Geometric methods for classical field theories

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Classical field theories can be eminently described in terms of differential geometry: classical fields are sections of a fibre bundle and the dynamics of the field are encoded in a certain differential form, called the Poincaré-Cartan form, on the first jet bundle.

We show how to extend this geometric description to include systems with constraints. In particular, we show first and foremost what it means for a field theory to be constrained. Secondly, we outline a geometric framework to deal with such theories, and we describe a simple example of such a system. This example has its roots in the theory of Cosserat media, and incorporates aspects of mechanical nonholonomic systems.

In the second part, we then make the connection with the theory of Lie groupoids. These objects show up in the discretisation of classical mechanical systems and field theories, and their preservation seems to be important in the construction in good numerical integration schemes.

References
