Lax algebras, Approach spaces and its applications

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In mathematics, one often seeks a unified way to describe different objects. The Eilenberg-Moore or strict algebras for a certain monad (a functor and two natural transformations) allow us for example to describe similarly the category $\text{Set}$ and the category $\text{CompHaus}$ of compact Hausdorff spaces. We obtain these categories because we actually describe the equality in $\text{Set}$ or the unique convergence of ultrafilters in $\text{CompHaus}$. In order to allow an ultrafilter to have more convergence points, we have to ‘relax’ the axioms of strict algebras. In stead of demanding equalities, we now allow inequalities. Using the same monads, we obtain in this way the category $\text{ord}$ of ordered spaces and the category $\text{Top}$ of topological spaces. The category $\text{Ap}$ of approach spaces, a numerical generalization of topological spaces, is recently obtained as a category of lax algebras for a new monad. It still is not possible to describe the category $\text{Lip}$ of Lipschitz spaces in the same way. We introduced a new monad and obtain a bigger category $\text{SLip}$ as a category of lax algebras. Using the same terminology, we can give a new characterization of $\text{Ap}$, $\text{Top}$ (a coreflective subcategory of $\text{Ap}$ and $\text{SLip}$), $\text{Ord}$ and $\text{Met}$ of metric spaces.
References


