Incidence geometry from an algebraic graph theory point of view

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An incidence geometry of rank $n$ consists of $n$ sets of objects $P_1, \ldots, P_n$, together with a symmetric incidence relation between elements of different types. A standard example is the projective geometry associated with $V(n+1, q)$, the vector space of dimension $n+1$ over a finite field of order $q$. Here $P_i$ with $1 \leq i \leq n$ is the set of $i$-dimensional subspaces and incidence is just (symmetrized) inclusion. A typical question about this geometry is:

*How many $k$-dimensional subspaces can we find such that the intersection of every two elements has dimension zero?*

In my research, I try to tackle these problems from a graph theory point of view. A graph $\Gamma$ is a pair $(V, E)$, where $V$ is a finite set of objects known as *vertices*, and $E$ is a set of unordered pairs of vertices, known as *edges*. If $\{v_1, v_2\}$ is an edge of $\Gamma$, then $v_1$ and $v_2$ are said to be *adjacent vertices*. A clique in a graph is a subset $C$ of vertices, such that every two elements of $C$ are adjacent. The previous problem can now be translated into a graph theory problem as follows:

*Let $\Gamma$ be the graph with the $k$-dimensional subspaces of $V(n+1, q)$ as vertices, and with two adjacent if they intersect in a subspace of dimension zero. What is the largest possible size of a clique in $\Gamma$?*

I mainly use matrix techniques for these problems. With each graph $\Gamma = (V, E)$, a symmetric $(|V| \times |V|)$-matrix $A$ can be associated, known as the *adjacency matrix*. If $v_1$ and $v_2$ are adjacent, then $A_{v_1, v_2} = 1$, and if they are not, then $A_{v_1, v_2} = 0$. As $A$ is symmetric, it has real eigenvalues and an orthonormal basis of eigenvectors. These algebraic properties sometimes allow us to obtain very powerful yet completely combinatorial results. Many fundamental results in this area were obtained by the Belgian mathematician Delsarte.

In the presentation I will sketch some of the available techniques, and discuss some results in incidence geometry by my colleagues and myself.