Bohr's theorem for Beurling integer systems

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Abstract

Bohr's theorem is a theorem about (ordinary) Dirichlet series; it says that if $f(s) = \sum_n a_n n^{-s}$ converges somewhere and has bounded analytic extension to $\Re s > 0$, then the series converges uniformly for $\Re s > \varepsilon$, for every $\varepsilon > 0$. For general Dirichlet series $\sum_n a_n e^{-\lambda_n s}$ Bohr's theorem need not hold. Nonetheless, having this theorem is desirable; it is for example a necessity for having a satisfactory Hardy space theory of Dirichlet series.

In this talk we consider general Dirichlet series whose frequencies come from Beurling generalized number systems: $\lambda_n = \log \nu_n$, where $\mathcal{N} = (\nu_n)_{n \geq 1}$ is the sequence of Beurling generalized integers generated by a system of Beurling generalized primes $\mathcal{P} = (q_n)_{n \geq 1}$. We show that there exist Beurling number systems $(\mathcal{P}, \mathcal{N})$ for which Bohr's theorem holds for the class of associated Dirichlet series $\sum_n a_n \nu_n^{-s}$, and additionally, for which the Riemann hypothesis holds.

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