Functional inequalities on Lie groups

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Abstract

The presentation will be about a family of functional inequalities on the general setting of Lie groups, or in some cases just in the case of Carnot groups. These inequalities are well-known in the Euclidean space $\mathbb{R}^n$ and we will discuss their extension in the general setting; recall that:

$$\text{general Lie groups} \subset \text{homogeneous} \subset \text{graded} \subset \text{Carnot} \subset \{\mathbb{R}^n, \text{Heisenberg}\}$$

These inequalities include: the log- Sobolev, Nash and generalised Poincaré inequalities. Interestingly, restricting ourselves to the setting of Carnot groups, we also show the "semi-Gaussian" (or Gross-type) inequality which recovers the classical Gross inequality when the Carnot group is simply $\mathbb{R}^n$. In the case of the Heisenberg group $\mathbb{H}^n$, the appearing constant allows us to pass to infinite dimensions, which in turn gives rise to an infinite-dimensional Heisenberg group with the first stratum of infinite dimension and a probability measure on it. Under this measure we prove the generalised Poincaré inequality on $\mathbb{H}^n$.