On the dynamics of the $\times_p$ and $\times_q$ maps on the unit circle.

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Abstract

For every integer $n \geq 1$, denote by T_n the map $x \mapsto nx \bmod 1$ from the circle group $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ into itself. Let $p, q \geq 2$ be two multiplicatively independent integers. I will present an overview of Furstenberg's $\times_p$-$\times_q$ conjecture, which states that any continuous Borel probability measure on $\mathbb{T}$ which is simultaneously T_p- and T_q-invariant must be the Lebesgue measure on $\mathbb{T}$. Using Baire Category arguments, I will then show that generically, a continuous T_p-invariant probability measure μ on $\mathbb{T}$ is such that $(T_{q^n}\mu)_{n\geq 0}$ does not converge w^* to the Lebesgue measure on $\mathbb{T}$. This disproves Conjecture (C3) from a 1988 paper by R. Lyons, which is a stronger version of Furstenberg's rigidity conjecture.

The talk will be based on a joint work with Catalin Badea (Lille).