Multivariate geometric quantiles: PDE aspects, universality, and Kolmogorov's distance

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Abstract

The concept of *geometric quantiles and cdf* is one of the most popular approaches, in the statistical community, to extend the usual quantiles and cdf from $\mathbb{R}$ to $\mathbb{R}^d$. After introducing the notion of *geometric cdf*, this presentation will focus on some progress made recently in our understanding of this object. We will first explain how an arbitrary probability measure P on $\mathbb{R}^d$ can be recovered from its geometric cdf F_P^{g} through a (potentially fractional) linear PDE, and how, quite unexpectedly, this leads to different behaviours when d is odd or even. We then explain how, in fact, the geometric cdf is essentially unique in the class of admissible linear cdf assignements $P \mapsto F_P$. Finally, we will show that the geometric cdf characterizes weak convergence of probability measures in $\mathbb{R}^d$, satisies a Glivenko-Cantelli result, and provides a natural extension to $\mathbb{R}^d$ of Kolmogorov's distance, with explicit upper-bounds in terms of the total variation distance and Wasserstein distances.