About generalized polynomials on Lie groups

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Abstract

M. Fréchet proposed an alternative definition of polynomial functions in 1909. A polynomial of degree at most m can be seen as a continuous solution of the functional equation

$$\Delta_{h_1,\ldots,h_{m+1}} f(x) = 0 \quad \forall x, h_1, ..., h_{m+1} \in \mathbb{R}.$$

The continuity assumption is actually important since it was possible to build a solution that is everywhere discontinuous solution and unbounded on any measurable set of positive measure.

Over decades, many people tried to see what happens when the unknown function is defined on more general domains such as semigroups or groups. It appears that for some classes of groups, there may not be any nonconstant solutions. For instance, such a phenomenom happens when the group is semisimple or compact.

With S. Nicolay, we studied these equations on Lie groups from a local point of view and with very minimal regularity assumption. We will present in this talk results regarding the equation extended to distributions and give a concrete description of the space of solutions. This will also describe a rigidity property concerning the existence of local nonconstant solutions. Another related functional equation will also be presented and some links with the previous question will be established.