

Hilbert's 10th Problem and Decidability in Algebra and Number Theory

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Abstract

In 1900, David Hilbert posed a question at an international mathematics conference in Paris: Is there an algorithm that can determine whether a given polynomial equation with integer coefficients has an integer solution? The question became known as Hilbert's 10th Problem. Several decades later, it became increasingly clear that such an algorithm may never exist. This marked the start of a research area on the intersection of logic, algebra, and number theory: to determine which classes of problems from number theory and algebra are decidable (i.e. solvable by an algorithm) and which are undecidable.

During this talk, I will briefly discuss the history of Hilbert's 10th Problem and highlight some recent developments. The second part of the talk will focus on how questions surrounding Hilbert's 10th Problem give rise to interesting problems in number theory, and conversely, how classical theorems from algebra, number theory, and quadratic form theory have been used to investigate decidability questions.