

Flag-Transitive geometries with trialities and no dualities coming from Suzuki groups

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Abstract

The results presented in this talk are part of a broader program that intends to study external symmetries (i.e: correlations) of incidence geometries. A well known instance of such external symmetries is the notion of duality. Dualities appears in vector spaces, projective spaces but also in polytopes and many other geometric objects. They are symmetries of the underlying object that exchange two types of elements (points and hyperplanes, vertices and facets, ...). We want to investigate geometric objects that admit external symmetries that exchange three types of elements cyclically. Such symmetries are called trialities. We are particularly interested in geometries admitting trialities but no dualities, as this phenomenon seems rather peculiar.

Recently, Leemans and Stokes constructed an infinite family of incidence geometries admitting trialities but no dualities from the groups $PSL(2, q^3)$. Unfortunately these geometries are not flag-transitive. Here we present the construction of the first infinite family of incidence geometries of rank three that are flag transitive and have trialities but no dualities. These geometries are constructed using chamber systems of Suzuki groups $Sz(q^3)$ and the trialities come from field automorphisms. We believe similar techniques can be used to produce geometries with trialities but no dualities from many other groups. Projective linear groups and Ree groups would seem to be good candidates.