A sandpile model for the shuffle theorem

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Abstract

The shuffle theorem is one of the most famous results in algebraic combinatorics of the last decade. Conjectured in 2001 by Haglund, Haiman, Loehr, Remmel and Ulyanov and proved in 2016 by Carlsson and Mellit, this theorem provides an explicit combinatorial formula for the symmetric function ∇e_n; also known as the Frobenius characteristic of the diagonal coinvariants. In this work, we provide an entirely different combinatorial model for the same function, using recurrent configurations of the sandpile model on a certain class of graphs. In our formula, we use the classic *level* statistic (related to the number of grains in the configuration), and a new statistic we call the *delay*. The proof of our result is a bijective combinatorial one, relating sandpile configurations to labelled Dyck paths, as they appear in the shuffle theorem.