Partial modules and module categories

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Joint work with Eliezer Batista (Universidade de Santa Catarina)
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Abstract

An action of a group G on a set X associates to each $g \in G$ an automorphism (permutation) of X. A natural generalization of this widespread concept is the *partial action*: the map associated to g is now defined only on a subset of X, that we call the domain of g.

The linearized version of a partial action is called a partial module. It turns out that the category of partial modules of a group (or more generally, of a Hopf algebra) has a rich structure. First, it is *monoidal*, which means that it comes equipped with a monoidal product on the objects: the appropriate tensor product of two partial modules is again a partial module. Second, it is a *module category* of the category of global modules. Stated otherwise, the category of global modules has an action on the category of partial modules.

In this talk, I will gently introduce the notions of partial module, monoidal category and module category by giving concrete examples. After this, we will look at some recent results obtained in collaboration with Eliezer Batista and Joost Vercruysse. Notably, the module category perspective allows to interpret the *globalization procedure* of partial modules as the right adjoint of a natural action functor.