

Building pretorsion theories from torsion theories

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Abstract

Pretorsion theories are defined as “non-pointed torsion theories”, where the zero object and the zero morphisms are replaced by a class of “trivial” objects and a suitable ideal of morphisms respectively. Thus, the notion of pretorsion theory can be defined in any arbitrary category \mathcal{C} , starting from a pair $(\mathcal{T}, \mathcal{F})$ of full replete subcategories of \mathcal{C} where \mathcal{T} and \mathcal{F} consist of the classes of “torsion” and “torsion-free” objects, and whose intersection defines the class of “trivial objects”.

In this talk, we present two ways of obtaining pretorsion theories starting from torsion theories, so that many new examples of pretorsion theories can be given in pointed categories. After recalling some key background on torsion and pretorsion theories, we shall describe pretorsion theories coming from pairs of torsion theories. Lattices and chains of torsion theories are widely studied topics and they are the perfect framework for applying our result. Then, we shall show how to obtain pretorsion theories “extending” a torsion theory with a Serre subcategory. We shall also present a universal way of obtaining a torsion theory from a given pretorsion theory in additive categories. We shall discuss some applications in representation theory and in the framework of recollements of abelian categories.

This talk is based on a joint work with Francesca Fedele and Emine Yıldırım (University of Leeds, United Kingdom).

References:

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