Riemannian trust-region methods for strict saddle functions with complexity guarantees

Florentin Goyens
Université catholique de Louvain (Belgium)

goyensflorentin@gmail.com

Abstract

The difficulty of minimizing a nonconvex function is in part explained by the presence of saddle points, which slows down the algorithm. This also directly impacts the worst-case complexity guarantees. However, a number of nonconvex problems of interest possess a favorable structure for optimization, in the sense that saddle points can be escaped efficiently by an appropriate algorithm. This so-called strict saddle property has been extensively used in data science to derive good properties for first-order algorithms, such as convergence to second-order critical points. However, the analysis and the design of second-order algorithms in the strict saddle setting have received significantly less attention. We consider second-order trust-region methods for a class of strict saddle functions defined on Riemannian manifolds. These functions exhibit (geodesic) strong convexity around minimizers and negative curvature at saddle points. We show that the standard trust-region method with exact subproblem minimization finds an approximate local minimizer in a number of iterations that depends logarithmically on the accuracy parameter, which significantly improves known results for general nonconvex optimization. We also propose an inexact variant of the algorithm that explicitly leverages the strict saddle property to compute the most appropriate step at every iteration. Our bounds for the inexact variant also improve over the general nonconvex case, and illustrate the benefit of using strict saddle properties within optimization algorithms.

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