Generalized symmetry operators for the parabolic Dirac operator

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Abstract

The main object of study in Clifford analysis is the Dirac operator $\partial_{\underline{x}}$, which arises when factorizing the Laplace operator $\Delta_{\underline{x}}$ and is constructed in such a way that no fractional derivatives are needed. In order to achieve this, one can use Clifford algebras, which are a generalization of quaternions. It is also possible to use Clifford algebras to construct a square root of the heat operator $H = -\Delta_{\underline{x}} + \partial_t = D^2_{\underline{x},t}$. The resulting operator $D_{\underline{x},t}$ is called the parabolic Dirac operator, and forms the main subject of this talk. More specifically, we will discuss the generalized symmetry operators D_0 of $D_{\underline{x},t}$, i.e. differential operators D_0 that satisfy $D_{\underline{x},t}D_0 = D_1 D_{\underline{x},t}$ for some differential operator D_1. Finally, we will elaborate on the algebraic structure generated by a subset of these generalized symmetry operators, which turns out to form a $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded Lie algebra.