Generation of vertex-girth-regular graphs

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Abstract

This talk is centered around a problem that naturally bridges two important areas of graph theory: the cage problem and vertex-transitive graphs (see for example the survey [1]). For integers v, k, g and λ, a vertex-girth-regular(v, k, g, λ) graph is a graph on v vertices in which every vertex has k neighbours, the length of the shortest cycle is equal to g and every vertex is contained in exactly λ shortest cycles. We consider the problem of determining the smallest such graphs for fixed k, g and λ. We discuss an algorithm that can effectively generate all vertex-girth-regular(v, k, g, λ) graphs for given integers v, k, g and λ and use this to determine the smallest such graphs for small parameters. This talk is based on joint work with Robert Jajcay and István Porupsánszki [2].

References

[1] G. Exoo and R. Jajcay. Dynamic cage survey. *Electron. J. Combin.*, DS16:48, 2008.

[2] R. Jajcay, J. Jooken and I. Porupsánszki. On vertex-girth-regular graphs: (Non-)existence, bounds and enumeration. *arXiv preprint arXiv:2408.14557*, 2024.