(Almost) arithmetic BMW groups

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Abstract

Let T_1 and T_2 be two regular trees of degrees $d_1, d_2 > 1$, respectively. A **BMW group** is a subgroup $\Gamma \leq \mathrm{Aut}(T_1) \times \mathrm{Aut}(T_2)$ acting freely and transitively on the vertices of $T_1 \times T_2$, the Cartesian product of the two trees.

By definition, a BMW group Γ is **arithmetic** if the closures of its projections on $\mathrm{Aut}(T_1)$ and $\mathrm{Aut}(T_2)$ are both rank 1 simple algebraic groups over local fields. One can construct infinite families of arithmetic BMW groups using quaternion algebras over global fields.

A quaternionic BMW group has the particularity to possess an infinite number of finite quotients of type PSL_2 or PGL_2 over specific finite fields, depending on the quaternion algebra. In this talk, we will study peculiar examples of BMW groups which are not arithmetic but have exactly the same finite PSL_2 and PGL_2 quotients as quaternionic groups. We will explain this coincidence and why we should enlarge the study of arithmetic BMW groups to those similar to the given examples.