Newsletter

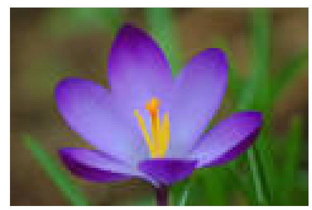
BELGIAN MATHEMATICAL SOCIETY

102, March 15, 2015

Comité National de Mathématique CNM

NCW Nationaal Comite voor Wiskunde



Newsletter of the Belgian Mathematical Society and the National Committee for Mathematics

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Contents

From the editor

Welcome to the March edition of our Newsletter.

Have a nice spring time!

Regards,
Françoise

1 News from the BMS & NCM

1.1 Follow us on twitter and like us on facebook!

Following some discussion in the Steering Committee, the Board agreed that we should improve our presence in the social media. You can follow `BelgianMathS` on twitter and tweet announcements or other interesting information to @BelgianMathS.

We also have a facebook page: https://www.facebook.com/BelgianMathS. Again, this page is your page! Please help us to keep it up to date and interesting by sending us nice links and information.

Thanks to Yvik Swan and Renaud Lambiotte for taking care of our social media!

New logo, banner, website,...

Do you think we should improve our looks and our website? Do you have experience with designing websites or logos? Or you know someone who does?

Share your thoughts with the Steering Committee! Mail Philippe Cara at <pcara@vub.ac.be>.

2 Meetings, Conferences, Lectures

2.1 March 2015

Young Mathematicians Colloquium

KU Leuven Kulak

March 16, 2015

Dear colleagues,

It is our great pleasure to announce the second edition of the Young Mathematicians Colloquium.

This meeting is an excellent opportunity for PhD students and young researchers in mathematics of Belgium and Nord-Pas-de-Calais to meet each other. The colloquium takes place at KU Leuven Kulak (Kortrijk) on Monday, March 16, 2015.

The speakers of this second edition are:
- Ann Dooms (VU Brussels)
- Etienne Ghys (ENS Lyon)
- Alain Valette (University of Neuchatel).

Just as last year, there will be a round table, this time about "Career options after a PhD in mathematics".

More details are available on the website http://www.mathconf.org/ymc2015 where you can also register for this meeting.

On behalf of the organizers,

Jonas Deré

2.2 May 2015

Genericity and small sets in analysis

Esneux (Domaine du Rond-Chêne)

May 26-28, 2015

See the poster at the end of this Newsletter.

More information:
• F.Bastin@ulg.ac.be
• the web site at http://www.afo.ulg.ac.be/fb/meeting/genericity/

3 Job announcements

No jobs this time but please send us your job openings for the next issue!

Next Newsletter will appear on May 15. The deadline for contributions is May 10. Contact Françoise Bastin <F.Bastin@ulg.ac.be>.

4 PhD theses

Group representations: idempotents in group algebras and applications to units
by
Inneke Van Gelder, Vrije Universiteit Brussel, FWO-Vlaanderen

Thesis supervisors: Prof. Dr. Eric Jespers and Prof. Dr. Gabriela Olteanu

Public defence: Friday March 27th, 2015 at 16h in room D.2.01 (promotiezaal) of the VUB campus Etterbeek.

Summary. In this thesis, we study the group $\mathcal{U}(RG)$ of units of RG, where R is the ring of integers of a number field F. In particular, we consider the group $\mathcal{U}(\mathbb{Z}G)$.

First, we investigate the primitive central idempotents and the Wedderburn decomposition of group algebras FG, with F a number field and G a strongly monomial group. Next, we focus on a complete set of matrix units in the Wedderburn components of $\mathbb{Q}G$ and $\mathbb{F}G$, with $\mathbb{F}$ a finite field, for a class of finite strongly monomial groups G containing some metacyclic groups.

We will also classify the finite groups G for which, given a fixed abelian number field F, the Wedderburn components of FG are not exceptional.

Thereafter, we study the central units $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$ for finite groups G. We construct generalized Bass units and show that they generate a subgroup of finite index in $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$, for finite strongly monomial groups G. For a specific class within the finite abelian-by-supersolvable groups G, we can even describe a multiplicatively independent set (based on Bass units) which generates a subgroup of finite index in $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$. For a different class of finite strongly monomial groups, containing some metacyclic groups, we construct such a set of multiplicatively independent elements starting from generalized Bass units.

Finally, we combine all results to construct a generating set of $\mathcal{U}(\mathbb{Z}G)$ up to finite index, for some classes of finite groups G.

5 History, maths and art, fiction, jokes, quotations ...

Espace Livres et Création, nouveautés l'Arbre de Diane (the two books are in French): see at the address http://espace-livres-creation.be/editeur/arbre-de-diane/

One of the two books, "Les mathématiques sont la poésie des sciences", is by C. Villani.

And, with great pleasure as always: find here some reviews by A. Bultheel, and also some pi-trivia by P. Levrie (celebration of pi-day ... very special this year: 3/14/15)

Genericity & Small Sets in Analysis

May 26-28, 2015

Rond-Chêne domain, Esneux (near Liège, Belgium)

Short introductory courses will be given by:

Frédéric Bayart (Clermont-Ferrand, France)
Juan Benigno Seoane Sepúlveda (Madrid, Espagne)

The list of speakers includes:

Richard Aron, Zoltán Buczolich, José Alberto Conejero, Robert Deville, Céline Esser, Yanik Heurteaux, Gustavo Adolfo Muñoz Fernández, Étienne Matheron, Marina Murillo-Arcila, Stéphane Seuret

Organizers:

Françoise Bastin, Frédéric Bayart, Stéphane Jaffard

Prevalence
Lineability
Porosity
Quasi-sure

Registration and additional informations can be found on the website of the conference:

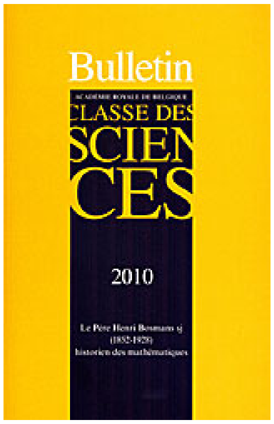
http://www.afo.ulg.ac.be/fb/meeting/genericity/index.php?p=home

Haar-null
Algebrability
Almost sure
Baire categories
Gaussian null
Sigma-porous sets

GdR Analyse multifractale

GdR Analyse fonctionnelle, harmonique et probabilités

Le père Henri Bosmans sj (1852-1928) *Michel Hermans and Jean-François Stoffel (eds.)* Bulletin de la Classe des Sciences, 6e série, Tome XXI, Académie Royale de Belgique, 2010, ISSN 0001-4141, 320 pp.

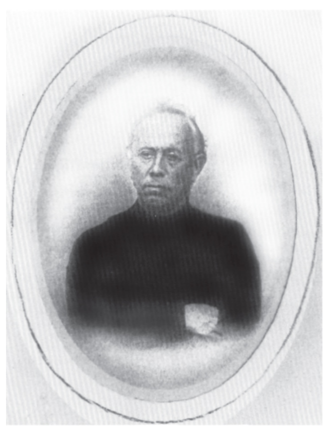


In 2006 in Brussels and in 2008 in Namur, some study days were organized devoted to Henri Bosmans and the proceedings are captured in this issue of the *Bulletin de la Classe des Sciences*.

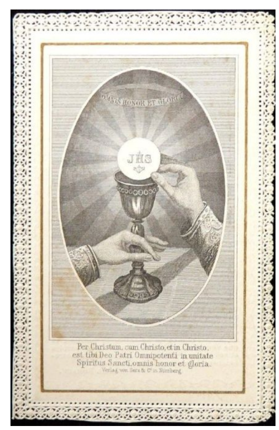
Bosmans was a Jesuit priest, and a mathematician. He published about 300 papers, mainly on the history of mathematics. He is important for our Belgian Mathematical Society because the *Mathematical circle* created in 1921 and chaired by *Théophile De Donder* was renamed as the *Belgian Mathematical Society* (or *Société Mathématique de Belgique* as it was probably called in those days) and Henri Bosmans became the president from 1923 till 1925.

Bosmans was born on 7 May 1852 from a father in the military, then stationed in Mechelen, and a mother from Ghent. He studied philosophy and entered the Jesuit order in 1871. After his theology studies in Leuven, he was ordinated on 8 September 1885, and was a mathematics professor in the *Collège Saint Michel* in Brussels from 1887 on for the rest of his life. In the minutes of the meeting of the BMS we find[1]:

> *Séance du 25 février 1928. En ouvrant la séance, M de la Vallée Poussin, président, rend un hommage ému à la mémoire du R.P. Bosmans, ancien président de la Société pour la période 1923–1925. Il rappelle la longue carrière professorale au Collège St Michel où il forma une pléiade d'élèves futurs ingénieurs et officiers. Il évoque les remarquables travaux sur l'histoire des mathématiques et le labeur infatigable de l'historien qui, presque aveugle, ne continuait pas moins ses patientes recherches. Il était d'ailleurs assidu de nos réunions mensuelles.*



Henri Bosmans (1852-1928)



Souvenir of his ordination 1885

His scientific career came along with his arrival in Brussels. It started by questions sent to *L'intermédiaire des mathématiciens* which he answered under an alias *H. Braid*. Around the turn of the century, he started writing bibliographical papers and engaged in the history of sciences and mathematics in particular. This earned him national and international recognition. Because of a disease, he was practically blind since 1913. This moved him away from teaching towards more historical research. Notwithstanding his vision problem, he meticulously copied many original sources that are still very important today for historians. He published books on fellow Jesuits: *Ferdinand Verbiest, directeur de l'observatoire de Péking*, *L'œuvre scientifique de Matthieu Ricci S.J.*, *Le jésuite mathématicien anversois, André Tacquet*, *Le géomètre Jérôme Saccheri S.J.* and *L'œuvre scientifique d'Antoine Thomas de Namur S.J.* A complete bibliography is in these proceedings compiled by Albrecht Heeffer, but it is also available online[2].

The booklet contains much more details about the Bosmans family, the education and the teachers of Henri Bosmans, his network (collaborators, librarians, bollandists, scientific societies).

[1] Le Père Henri Bosmans et la Société Mathématique de Belgique, by Michel Hermans sj and Paul van Praag

[2] `http://logica.ugent.be/albrecht/bosmans.php`

Collège Saint Michel 1920

One contribution is devoted to Paul Mansion (1844-1919) who was Bosmans's teacher in Ghent and with whom he kept in correspondence in his further career. Letters from Mansion to Bosmans, now in the Jesuit archives, are discussed in more detail in a separate paper of these proceedings.

Paul Mansion

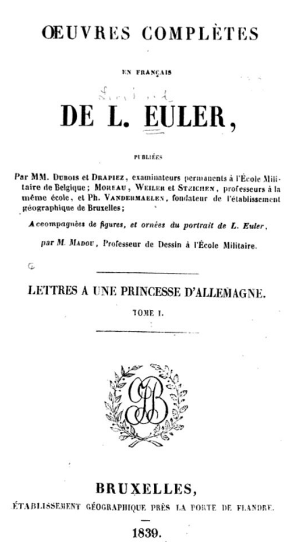
Another one concerns the Jesuit Girolamo Saccheri (1667-1733) who is mainly remembered for his work on non-Euclidean geometry, long before Bolyai and Lobachevsky. This is thanks to work done in Italy and Belgium. Beltrami (Turin) and Mansion (Ghent) saved his work from oblivion. Later, Saccheri and his work was further analysed by another Italian-Belgian twosome: Corrado Segre (1863-1924) in Turin and Bosmans in Brussels,

J. Mawhin discusses in his paper the Belgian attempt in 1839 to publish the complete works of Euler in French. Finally five volumes were published[3]. Bosmans has devoted a careful analysis on this work and its editors.

Most of Bosmans's biographical work is devoted to scientists that belonged to his own Jesuit Order. Clearly because he had access to the archives, but also just because many of the scientists happened to be Jesuits. The connection between Jesuits and China is well known. Ferdinand Verbiest (1623-1688) is one the exponents. But Bosmans also wrote about the exchange of knowledge and information from and to China. The sources he used (and missed) are exposed in another paper.

Two papers are devoted to the methods used by Bosmans and historiography in general at the beginning of the 20th century (in Belgium). Another one describes the archives and their importance. After his death the archives were split into the printed work, that mainly concerned the history of mathematics and that went to a Jesuit College in Egenhoven (unfortunately destroyed by a fire in 1940) and the manuscripts mostly about the Jesuit Order that went to Saint Michel in Etterbeek.

A. Bultheel



ŒUVRES COMPLÈTES

EN FRANÇAIS

DE L. EULER,

PUBLIÉES

Par MM. Dubois et Drapiez, examinateurs permanents à l'École Militaire de Belgique; Moreau, Weiler et Steichen, professeurs à la même école, et Ph. Vandermaelen, fondateur de l'établissement géographique de Bruxelles;

Accompagnées de figures, et ornées du portrait de L. Euler,

par M. Madou, Professeur de Dessin à l'École Militaire.

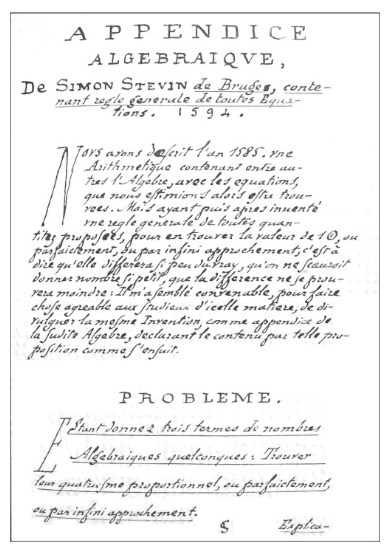
LETTRES A UNE PRINCESSE D'ALLEMAGNE.

TOME I.

BRUXELLES,

ÉTABLISSEMENT GÉOGRAPHIQUE PRÈS LA PORTE DE FLANDRE.

1839.



APPENDICE
ALGEBRAIQVE,
De Simon Stevin de Bruges, contenant regle generale de toutes Equations. 1594.

Nous avons descrit l'an 1585. vne Arithmetique contenant entre autres l'Algebre, avec les equations, que nous estimions alors estre trouvees. Mais ayant puis apres inuenté vne regle generale de toutes quantitez proposees, pour en trouver la valeur de 10, ou parfaictement, ou par infini approchement, c'est à dire qu'elle differera si peu du vray, qu'on ne scauroit donner nombre si petit, que la difference ne se trouvera moindre: Il m'a semblé convenable, pour faire chose agreable aux studieux d'icelle matiere, de divulguer la mesme Invention, comme appendice de la susdite Algebre, declarant le contenu par telle proposition comme s'ensuit.

PROBLEME.

Estant donnez trois termes de nombres Algebraiques quelconques: Trouver leur quatriesme proportionnel, ou parfaictement, ou par infini approchement.

Explica-

Bosmans manuscript

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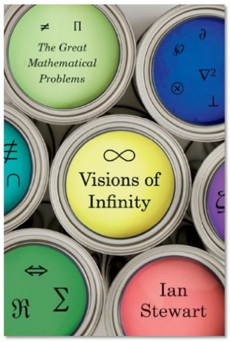
[3]Now available as Google books. See http://eulerarchive.maa.org/pages/E786.html.

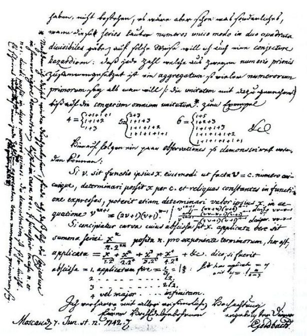
Visions of infinity. The Great Mathematical Problems, by *Ian Stewart*. Basic Books, 2013, ISBN 978-04-6502-240-3 (hbk), 352 pp.



Ian Stewart

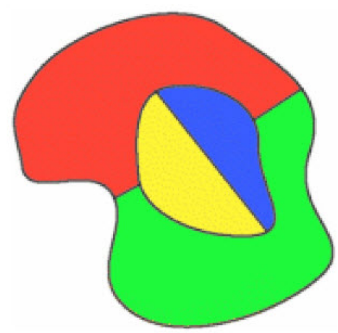
Ian Stewart is a professor of mathematics at Warwick University, well known for his many books on popular science. In this recent book he introduces the reader to the *Great Problems of Mathematics* (with capitals!). These are the ones that were often formulated as conjectures that gained fame because they remained unsolved for many years or that are still not answered in the positive or negative and that are still worth a million dollar. Since these usually generated a lot of mathematical progress, the answer whether the conjecture is true or false is not really the most important objective, but the new proof techniques and the whole new body of mathematics needed becomes a driving force for the development of mathematics and the true reason why prestigious prizes are rewarded for their proofs. And we all know several of these like Fermat's last theorem and the four colour problem. You're too late if you want to start solving those but the Riemann conjecture, and the P/NP problem for example are still open.

It is the intention of the author to describe the origin, the history, and the development of some of these really big questions that have driven mathematics in new directions, hence illustrating their importance for mathematics and science in general. It is not easy, but Stewart at least tries, with some success I must say, to transfer the ideas without the plethora of formulas and difficult mathematical concepts that one would expect. What counts are the notions, not the notation as Gauss once remarked. Obviously some of the more technical details have to be skipped, and while the ideas of the early chapters are easy to transfer, it becomes much more difficult, if one is not familiar with some more advanced mathematics, to grasp the millennium problems of the Clay Mathematics Institute that are introduced in the later chapters.



Goldbach's letter to Euler

π transcendental



4 colours are necessary

So after an introductory chapter, Stewart takes us on a journey along the history of the great challenges that mathematics lived through, or that are still marinating, waiting for a solution. Most enlightening is to see how he illustrates that a breakthrough of some tough problem is triggered by some advance in an, at first sight, unrelated piece of mathematics. Stewart's advise: if you can't solve it, put it aside, do something else, and come back later.

The first topic is the *Goldbach conjecture* (1742): every integer $n > 2$ is the sum of 2 primes. Everybody believes it to be true, but it remains unproven. Stewart takes his time to introduce prime numbers, their history and their computation. This pays because several of the subsequent problems are related to prime numbers as well. He explains what has been shown and what is not, how it has been linked to the Riemann conjecture and the open problems left for the present and future generations.

Squaring the circle is a problem that dates back to the Greeks and is clearly linked to π. This is a nice illustration of how an obviously geometric problem, is reformulated as an algebraic one, solving polynomial equations, leading to (complex) analysis, when one wants to prove that π is irrational and transcendental. Much more recent is the *four colour problem*. It was first formulated by a South African law student in 1852. He asked his brother's advise. The brother was studying mathematics at the University College in Londen where he asked A. De Morgan and the latter exposed it to the math community. Again, the solution is not important for cartography because there are many other reasons to choose a colour for the map, but it started research is networks and graphs, and it has been generalized to colour problems on much more complex topological surfaces. It was finally proved in 1976 by Appel and Haken and it revolutionized the concept of a proof, since it was the first time that a proof relied on the verification by a computer of many cases to which the problem was reduced. Too many to be checked by humans.

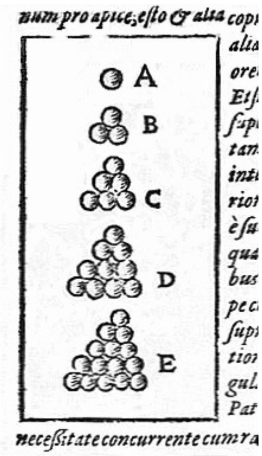
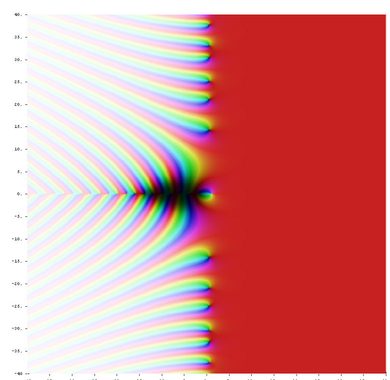


image from Kepler's snowflake book

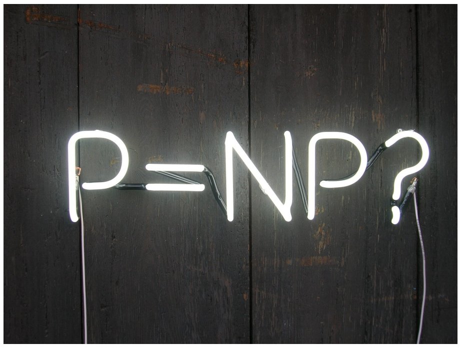
Another famous problem is known as the *Kepler conjecture*. Although the original version of 1611 by Kepler appeared in a booklet on 6-pointed snowflakes, it is related to sphere packing basically asking how dense equal spheres can be packed. Every grocer knows how to mount oranges in a pyramid, but it took 387 years for a proof to be found. Again, a computer was needed to solve the global optimization problem. A formal proof avoiding the computer is still an ongoing project. The *Mordell conjecture* (1922) was proved by Faltings in 1983. It is about Diophantine equations but has a geometric formulation stating that a curve of genus $g > 1$ over $\mathbb{Q}$ has only finitely many rational points. Stewart uses this on his path towards *Fermat's last theorem*, since one may start from Pythagoras equation $x^2 + y^2 = z^2$ with integers to the equation of a circle $(x/z)^2 + (y/z)^2 = 1$ with rational numbers and subsequently to a generalization where the circle is replaced by an elliptic curve. Stewart's account of the human aspects of doubt and fear of failure or err when Wiles was working on his proof of FLT has some remote reminiscences of a mathematical thriller.

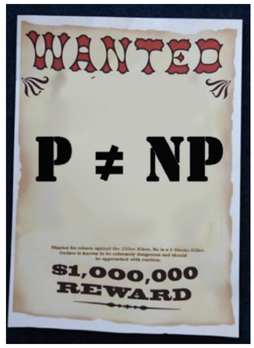

Riemann ζ function

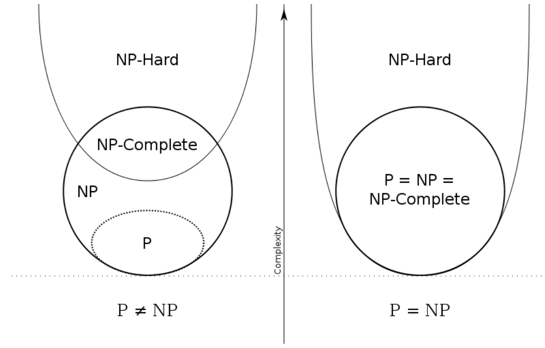
Although Poincaré got the award of the Swedish King Oscar II in 1887, he did tot really solve the originally posed *three body problem* that was suggested by Mittag-Leffler. Nowadays there are numerical techniques to solve equations with a chaotic solution approximately, rigorous proofs and many questions remain unanswered. The answers are directly related to fundamental questions about the stability of our solar system.

Back to prime number distribution with the *Riemann hypothesis* (1859). Again number theory is lifted to complex analysis in the study of the ζ-function (Stewart needs quite some pages to come to this point). It is explained how this leads to the conjecture that the zeros of the ζ-function are on the critical line $x = 1/2$ in the complex plane. This is the most famous open problem in mathematics today. It survived Hilbert's 1900 unsolved problems and it is reformulated as one of the Clay millennium problems. Most mathematicians believe it to be true as numerics seem to indicate but a proof is still missing.

The other six millennium problems are the subject of the following 6 chapters, in which the mathematics that Stewart needs become tougher. But where the going gets tough, the tough get going. The *Poincaré conjecture* (1904) was solved by Perelman in 2002, but because it took 8 years for the math community to verify his proof, the eccentric Perelman, totally disappointed with that situation, has withdrawn from mathematics and refuses all contact with the media. He declined the EMS prize (1996), Fields Medal (2006) and the Clay millennium prize (2010).

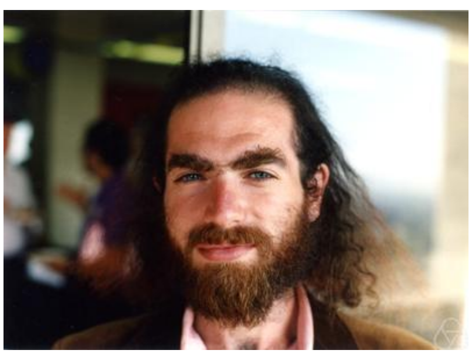






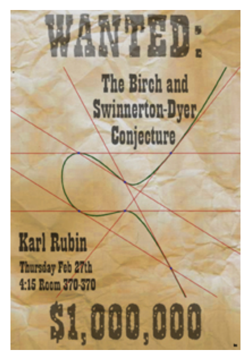
The *P/NP problem* is still open and the outcome is uncertain: are hard problems such as the traveling salesman problem solvable with polynomial time algorithms? The answer to this question seems to be NP-hard itself. P stands for polynomial time complexity, NP means non-deterministic polynomial time complexity, and NP-complete are the hardest problems from class NP. If one can solve these, then any other NP problem can be solved in polynomial time.

Solving the *Navier-Stokes equation* is a problem from applied mathematics. Can one verify that the small changes made by numerical procedures don't miss some turbulent solution because the approximation is not fine enough. In January 2014, Otelbaev claims to have solved this problem. The proof is at the moment (March 2014) still under dispute but probably flawed. The *mass gap hypothesis* relates to quantum field theory of elementary particles. These quantum particles have a nonzero lower bound for their mass even though the waves travel at the speed of light. In relativity theory, the mass would be zero. The *Birch-Swinnerton-Dyer conjecture* is another millennium problem about rational solutions of certain elliptic curve equations. Finally the *Hodge conjecture* connects topology, algebra, geometry and analysis to be able to say something about algebraic cycles on projective algebraic varieties.



G. Perelman (1993)

M. Otelbaev



BSD conjecture

Although Stewart tries very hard to introduce the unprepared reader to the problems and the techniques for the latter four problems, the much more advanced mathematical needs make these chapters much harder to read than the earlier ones. As a conclusion, he gives his own opinion of what will and what will not be proved in the (near) future. Just in case the reader gave up on the Riemann hypothesis and is looking for inspiration to find another really challenging problem, Stewart provides a list of 12 somewhat less known open questions that are as yet unsolved.

Stewart's entertaining style, his meticulous sketching of the historical context, his sharp analysis of the importance of the problems and their consequences, his broad insight on the wide spectrum of mathematics and his understanding of the human behind the mathematician, struggling for solutions and recognition, each one with his own character, makes this book, like his other books, a very interesting read. Some of the less mathematically experienced readers may not reach the end, but whatever can be assimilated is worthwhile.

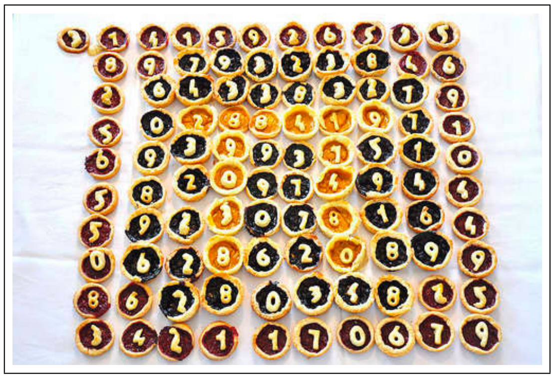
Adhemar Bultheel

pi-trivia



Did you know that . . .

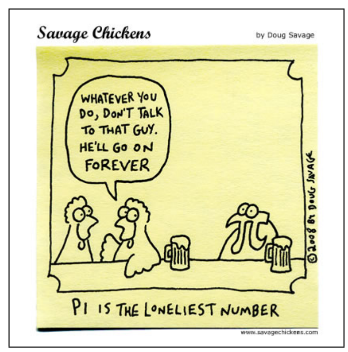
- . . . today is π-day?
Why? Because in America they write 3/14 for the date of today, March 14, and 3.14 is an approximation to the number π.
It's a very special π-day today: 3/14/15. And since π equals 3.1415926 . . ., you see that you should start eating pie at nine twenty six.



Enjoy this special π-day, because the next one will be in one hundred years.
- . . . the number π (still) is the ratio between the circumference of a circle and its diameter?
- . . . the first 500 decimal digits of π are given by:

3.141592653589793238462643383279502
88419716939937510582097494459230781
64062862089986280348253421170679821
48086513282306647093844609550582231
72535940812848111745028410270193852
11055596446229489549303819644288109
75665933446128475648233786783165271
20190914564856692346034861045432664
82133936072602491412737245870066063
15588174881520920962829254091715364
36789259036001133053054882046652138
41469519415116094330572703657595919
53092186117381932611793105118548074
46237996274956735188575272489122793
818301194913

- . . . March 14 is not the only day of the year that mathematicians especially like?
π can also be celebrated on July 22, called π approximation day, since $\frac{22}{7} = 3.142857$ is a very good rational approximation to π, and has been used as such for a very long time.
There is of course also τ-day, for the non-believers, on 6/28 ($\tau = 2\pi$).
And why not e-day? Named after Euler's constant $e = 2.718281828\ldots$, and celebrated on July 2.
This year we also have a Pythagoras day, on September 12 (9/12/15), because 9, 12 and 15 form a Pythagorean triple:
$$9^2 + 12^2 = 15^2$$
These three numbers are the sides of a right triangle!
If you're not free on that day in September, you get a second chance to celebrate this on December 9.
- . . . π is an infinitely long, non-repeating decimal number? Hence you cannot write the number π as a fraction with a whole number as numerator and as denominator.



- . . . instead of learning the digits of π by heart, you can do the same with the numbers of the sequence staring with 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2,. . . ? These numbers determine the value of π completely, since they can be used to generate what is called the simple continued fraction for π:

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \ldots}}}}}$$

and so on. You get rational approximations to π by terminating this fraction at some point, for instance after the 7:

$$3 + \frac{1}{7} = \frac{22}{7}$$

You get a very good (small) approximation if you stop just before the big denominator 292. The result is: $\frac{355}{113} = \underline{3.141592}92035\ldots$, sometimes called Zu's fraction, after the Chinese mathematician Zu Chongzhi (429-500).
And no, there's no repetition in this sequence either. You can find the first 20 000 000 terms of this sequence on the webpage of Simon Plouffe.

- … in 2014 a new world record Calculating-as-many-digits-of-π-as-possible has been set by someone going under the name houkouonchi who wants to remain anonymous? A total of 13 300 000 000 000 decimal digits were computed. The computation took 208 days, and the calculation was done using the following formula found by the Chudnovsky brothers:

$$\frac{1}{12\pi} = \sum_{k=0}^{\infty} \frac{(-1)^k (6k)!(13591409 + 545140134k)}{(3k)!(k!)^3 640320^{3k+3/2}}$$

  It gives 14 extra digits per calculated term.
- … in Canada pi^2 is a much used unit? It stands for pied carré, or square feet. (Thanks, Cris, for letting me know!)
- … the trophee one can win at the "Wiskunnend Wiske"-competition (organised by the VUB for secondary schools) looks like this: (Thanks, Sabien, for sending me this!)

- … we can use what we know about the number π to prove that the number of primes is infinite? To do this we use the following formula due to Euler:

$$\frac{\pi}{4} = \frac{3}{4} \cdot \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{13}{12} \cdot \frac{17}{16} \cdot \frac{19}{20} \cdot \frac{23}{24} \cdot \frac{29}{28} \ldots$$

  The numerators at the right hand side are the odd primes, and the corresponding denominator is the multiple of four closest to the numerator.
  Let us assume that the number of primes is finite. Then there's only a finite number of factors in the product at the right hand side. Hence the expression at the right hand side is a rational number. As a consequence the number π would be rational too, and we know that this isn't the case. (The decimal digits of a rational number are repeating.)
- … there's a song about a human π-calculator? It's in Dutch, sung by Drs. P. The title is Greek Tango, and you can enjoy it here: `https://www.youtube.com/watch?v=OyfIkFyn_Co`
- … there are π-related Apps? For instance a digital clock that searches for the current time and date in the decimal expansion of the number π (or e, or …):

- … the number π is omnipresent?

  (Thanks, Adhemar, for sending me this!)
- … the number π plays a part in the 2009 movie Night at the Museum 2: Battle of the Smithsonian? In it the protagonists are confronted with the following riddle: What number is at the heart of a Pyramid? To solve it, they have to ask some Einsteins for help:

  Strangely enough the answer is: 3.1415926...