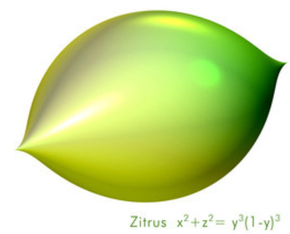
Newsletter

BELGIAN MATHEMATICAL SOCIETY

Comité National de Mathématique CNM

NCW Nationaal Comité voor Wiskunde

104, September 15, 2015



Newsletter of the Belgian Mathematical Society and the National Committee for Mathematics

ULB Campus Plaine, C.P. 218/01,
Bld du Triomphe, B-1050 Brussels,
Belgium

Website: http://bms.ulb.ac.be
Newsletter: F.Bastin@ulg.ac.be
Tel. F. Bastin, ULg(32)(4) 366 94 74

Contents

From the editor

Welcome to this September Newsletter! Have a nice semester!

Regards,
Françoise

---

1 News from the BMS & NCM

Important date:

Wednesday November 18th, 2015

General Assembly of the BMS

All members of the **Belgian Mathematical Society** are invited to the **General Assembly** in Leuven on Wednesday November 18th, 2015 at 4 pm. During this assembly a new executive committee will be elected. Activities of the past year will be commented and future projects will be presented.

The General Assembly will be followed by a lecture by Prof. Dr. Gert-Martin Greuel and an exclusive visit to the IMAGINARY exhibition.

Participation to this event is free, but it is compulsory to register at

http://wis.kuleuven.be/events/imaginary/bms-event-en.

There you will also find all practical information on the event.

The full program is as follows:

- 16:00 General Assembly of the BMS
- 16:30 Lecture Mathematics between Research, Application and Communication by Prof. Dr. Gert-Martin Greuel, founding father of IMAGINARY
- 17:30 Reception and visit to the IMAGINARY exhibition

Special thanks to Stefaan Vaes for organising this event.

About IMAGINARY

What do a lemon, a diabolo and a hummingbird have in common? You find an answer at IMAGINARY, an interactive mathematics exhibition that will travel through Belgium from September 28th 2015 onwards. The Belgian Mathematical Society invites all its members at a special IMAGINARY event in Leuven on Wednesday, November 18th 2015 at 16h00.

The first edition of IMAGINARY was organized in Germany, by the Oberwolfach Mathematics Institute. Since then, IMAGINARY became an international project, present on line at http://imaginary.org, and with past exhibitions in 30 countries attracting more than 1 million visitors.

In collaboration with all universities in Flanders and the BMS, the Flanders Mathematical Olympiad brings IMAGINARY to Belgium. The entrance to the exhibition is free. The exhibition itself is appealing to a broad audience. As a visitor you will see a series of marvelous, well documented gallery prints, a number of 3d objects printed specifically for this exhibition by partner Materialise, and interactive software to simulate, visualize and even play on large touch screens.

The organizers provide material for teachers and offer free guided tours at all locations to groups of secondary school pupils from ages 14 and on. These guided tours can be booked at http://www.imaginarymaths.be.

We hope to meet many Belgian mathematicians, and many more non-mathematicians, at the 2015-2016 IMAGINARY exhibition.

2 Meetings, Conferences, Lectures

2.1 October 2015

Academy Contact Forum "Coding Theory and Cryptography VI"

Friday, October 2, 2015, Brussels, Belgium

Poster at the end of this Newsletter.

Wandering colloquium

Getaltheorie in het vlakke land

Arithmétique en plat pays

Monday October 19, 2015, Ghent, Belgium

Poster at the end of this Newsletter.

Plonger des graphes dans des espaces vectoriels euclidiens… ou pas euclidiens!

by
Alain Valette (Université de Neuchâtel)

Thursday October 22, 2015, Brussels, Belgium

Poster at the end of this Newsletter.

2.2 November 2015

Une journée autour d'une même famille: Analyse et Probabilités

UMons, jeudi 5 novembre 2015

Information: see the poster at the end of the Newsletter.

3 Job announcements

No jobs this time but please send us your job openings for the next issue!

Next Newsletter will appear on November 20. The deadline for contributions is November 10. Contact Françoise Bastin <F.Bastin@ulg.ac.be>.

4 PhD theses

Hölder Continuity and Wavelets

Laurent Simons, University of Liège
Date and place: June 24, 2015, ULg

Thesis advisor and co-advisor: F. Bastin and S. Nicolay (ULg)

Abstract – directly from the thesis

There exist a lot of continuous nowhere differentiable functions, but these functions do not have the same irregularity. Hölder continuity, and more precisely Hölder exponent, allow to quantify this irregularity. If the Hölder exponent of a function takes several values, the function is said multifractal. In the first part of this thesis, we study in details the regularity and the multifractality of some functions: the Darboux function, the Cantor bijection and a generalization of the Riemann function.

The theory of wavelets notably provides a tool to investigate the Hölder continuity of a function. Wavelets also take part in other contexts. In the second part of this thesis, we consider a nonstationary version of the classical theory of wavelets. More precisely, we study the nonstationary orthonormal bases of wavelets and their construction from a nonstationary multiresolution analysis. We also present the nonstationary continuous wavelet transform.

For some irregular functions, it is difficult to determine its Hölder exponent at each point. In order to get some information about this one, new function spaces based on wavelet leaders have been introduced. In the third and last part of this thesis, we present these new spaces and their first properties. We also define a natural topology on them and we study some properties.

5 Miscellaneous

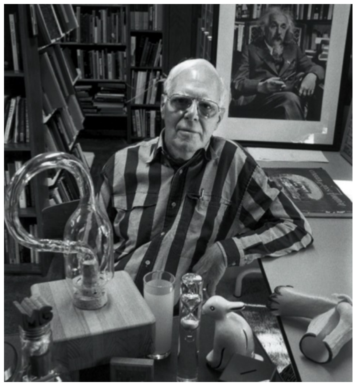
5.1 Francqui Prize 2015

Our Colleague **STEFAAN VAES** received the Francqui Prize! Warm congratulations to him!

You will find a text about the work of S. Vaes at the end of the is Newsletter.

6 History, maths and art, fiction, jokes, quotations ...

Knots and Borromean Rings, Rep-Tiles, and Eight Queens: Martin Gardner's Unexpected Hanging. by *Martin Gardner*. Cambridge University Press, 2014, ISBN: 9780521758710 (pbk), 288 pp.

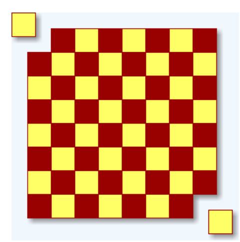


Martin Gardner, passed away in 2010 at the age of 95, is best known for popularizing mathematical puzzles. From 1956 to 1981 he wrote *Mathematical Games and Recreations*, a monthly column for *Scientific American*. In these columns he discussed brainteasers and other mathematical topics in an entertaining way. These columns were expanded and collected in 15 books and printed by diverse publishers.

In 2006, Martin Gardner agreed on a project together with the Mathematical Association of America and Cambridge University Press to revise all 15 books for *The New Martin Gardner Mathematical Library*. These new editions gave Gardner the chance to add new explanations, newer twists on old puzzles, recent developments and to expand the bibliographies.

This book, 4th in the series, is the new edition of *The Unexpected Hanging and Other Mathematical Diversions*, published by Simon & Schuster in 1969, which covers most of the 1961–1963 columns.

The book consists of 20 short chapters. In each chapter a problem is discussed, and the reader is challenged with some questions. At the end of each chapter, Gardner provides the answers to the questions and all updates are collected in an afterword, followed by a list of references.



Martin Gardner knows how to fascinate the reader when writing about the paradox of the unexpected hanging and logic, Borromean rings, knots and topology, the number *e* and much more. His famous problem about a checkerboard with two diagonally opposite corners removed is also included. The question is whether the remaining 62 cells can be completely covered by 31 dominoes. Another problem is the one of a tennis match. The reader knows that Miranda beat Rosemary in a set of tennis, winning six games to three. Five games were won by the player who did not serve. The question is who served first?

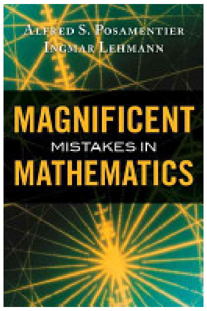
To give an idea about the wide range of topics, we list some of the titles of the chapters:

5. Scarne on Gambling
6. The Church of the Fourth Dimension
8. A Matchbox Game-Learning Machine
10. Rotations and Reflections
18. Curves of Constant Width
19. Rep-Tiles: Replicating Figures on the Plane

This book is an easy read with a lot of variation and without technicalities. Therefore it is recommended for both young and old, mathematicians and non-mathematicians.

I. Van Gelder

Magnificent Mistakes in Mathematics, 2013, Prometheus Books, ISBN 978-1-61614-747-1 (hbk), 198 pp. by *Alfred S. Posamentier and Ingmar Lehmann*

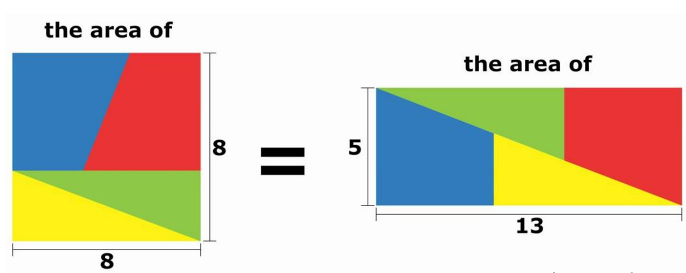


A.S. Posamentier & I. Lehmann

We all learn from our mistakes (and not only the mathematical ones). This is one of the reasons these authors wrote the book. By showing what kind of mistakes can be made in mathematics, and to what absurd conclusions that may lead, it is hoped that the reader understands better the rules of the game and be more careful in jumping to conclusions.

However, we may learn not only from our own mistakes. There have been many historical mistakes, made by leading mathematicians. The first chapter is a collection of such examples. Pythagoras was mistaken when he thought that nature could be completely explained with natural numbers and their ratios. There have been historical mistakes in the calculation of π, and many wrong attempts have been made to prove Fermat's last theorem, Goldbach's conjecture, or solve the 4 colour problem, and many other such famous problems. Galileo, Euler, Fermat, Legendre, Poincaré, Einstein, they all made mistakes and often in published papers. Gauss seems to be a glorious exception to this rule. No errors are known in his published papers. This chapter is an enumeration of summaries of these historical errors, although a complete book could be devoted to each of them, why and how the wrong conclusion was made and what kind of research this has started. For example, a wrong calculation of a notorious gambler *Chevalier de Méré* caused him to loose. He asked Pascal to explain what seemed to him a paradox, and the correspondence between Pascal and Fermat can be considered to be the start of probability theory. And we all know that the attempts to prove Fermat's last theorem has given rise to a many new mathematical results.

The subsequent chapters discuss arithmetical, algebraic, geometrical and statistical mistakes. Here we find many obvious errors that are commonly committed by students like division by zero, or violating the rules of distributivity ($\sqrt{a+b} = \sqrt{a}+\sqrt{b}$ or $\frac{a}{b}+\frac{c}{d} = \frac{a+c}{b+d}$ and the likes). Also jumping too soon to a general conclusion is common. Infinite sums, and working with ∞ often leads to false results. Of a somewhat different nature are the rounding errors of digital calculators or computers that may play dirty tricks on us. Sometimes wrong logic may lead to correct answers. Arguments are then needed to convince the student of the low score. All these are familiar and we are desperate when students persist sinning against them. Many examples of mistakes can however be reduced to the same error made under a slightly disguised form. So there is basically a lot of repetition which makes these chapters a bit dull from time to time. There are however also errors that are counter intuitive or that have some pitfalls and that are often used in quizzes or to astonish the innocent reader with an apparent paradox. For example suppose the earth is a perfect sphere. Put a rope around the equator and enlarge it by 1 meter. Then keep this longer rope at an equal distance above the surface. Can a mouse pass under the rope? Our intuition says no. However, computation results in a different result. The cat can pursue the mouse at the other hemisphere as well. There are the optical illusions and impossible figures in a geometrical context. A famous geometrical puzzle showing that $65 = 64$ popularized by Lewis Carroll is shown in the figure on the right. It has happened that mistakes were deliberately introduced as a prank. Martin Gardner presented in his April 1975 column of *Scientific American* a map that would require 5 colours, which turned out to be an April fools joke on his readers. Stories like this and the more recreational pitfalls will keep you reading to the end. A not always very deep, but a broad and diverse collection of examples. *Adhemar Bultheel*



The work of Stefaan VAES, Francqui Prize 2015

Alain Connes, Vaughan Jones, Sorin Popa, Alain Valette

18 May 2015

1 Brief biography

Stefaan VAES, a Belgian citizen, was born on Feb. 29, 1976. He got a Master degree in Mathematics in 1998 at KU Leuven (Belgium), a PhD in Mathematics in 2001 (under the supervision of Prof. Alfons Van Daele) at the same University, and a "Habilitation à diriger des recherches" in 2004 at Université Paris VII - Denis Diderot.

From 2002 to 2006, he was Research associate of the French CNRS, at the Institute of Mathematics of Jussieu, Paris. Since 2006, he was Assistant Professor and then Full Professor at the KU Leuven.

In 2006 he held a Peccot Chair at the Collège de France, Paris. He was an invited speaker at ICM 2010 in Hyderabad. He is an editor of *Journal of Functional Analysis, Transactions of AMS, Memoirs of AMS*, and *Forum of Mathematics-Sigma*

For more information, see his homepage:
`https://perswww.kuleuven.be/%7Eu0018768/`

2 Mathematical itinerary

In the early years of his career Stefaan Vaes revolutionized the operator algebraic approach to quantum groups by providing a stunningly simple definition of a locally compact quantum group. (see Section 3 below). Then in 2004 he moved into the study of the structure/classification of the II_1-factors of orbit equivalence (OE) relations arising from actions of groups on measure spaces and/or on other algebras. This has been the most active/exciting area of research in functional analysis in the last 10 years, involving a beautiful interplay of methods, from group theory (geometric, arithmetic, combinatorial, algebraic, Lie group theory, representation theory, bounded cohomology...), to ergodic theory and operator algebras.

There Stefaan Vaes showed his remarkable talent and obtained unprecedented, breakthrough results, that we analyze in section 4 below.

Vaes' complete list of papers can be found on his homepage. He has papers in the most prestigious journals, including the trilogy *Annals, Inventiones* and *Acta*.

Vaes work can be characterized by formidable technical power, uncanny creativity and a striking clarity of mind, a combination of qualities that enables him to carry out extremely hard and complex arguments. Somewhat unusually, these qualities are manifest in both his algebraic and analytic arguments.

3 Vaes' work on quantum groups

In a series of papers, with various collaborators (Vainerman, Baaj, Skandalis), between 2001-2003, Vaes has developed the theory of extensions of locally compact quantum groups, as bi-crossed products with cocycles. Historically, in the sixties, G.I. Kac had constructed the first Hopf algebra neither commutative, nor co-commutative as an extension of a finite group by the dual of a finite one. This remarkable example was indeed the starting point of the subject. A particular case is Drinfeld's quantum double construction. In his deep *Crelle* paper [Va05], Vaes has studied strictly outer actions of groups and quantum groups on operator algebras. An important motivation for this work is subfactor theory, where a key problem is to clarify which locally compact quantum groups can act strictly outerly on a factor, and on which kind of factor. Even in the case of classical locally compact groups the problem is quite difficult. A. Wassermann had proved that any compact Lie group acts strictly outerly (even minimally) on the hyperfinite II_1-factor. A first result of Vaes is that every locally compact quantum group acts strictly outerly on the hyperfinite II_1-factor. Another remarkable result is Vaes free product construction showing that every locally compact quantum group has a strictly outer action on the Shlyakhtenko free Araki-Woods type III_1-factor (not hyperfinite). In addition, Vaes obtains necessary and sufficient conditions for a locally compact quantum group to act strictly outerly on different Connes-types of factors.

4 Vaes' work in von Neumann algebras and ergodic theory

4.1 Framework

Most of Vaes work in these areas is on rigidity aspects in von Neumann algebras (II_1- *factors*) $L(\Gamma)$, $L^\infty(X) \rtimes \Gamma$, and orbit equivalence relations $\mathcal{R}_{\Gamma \curvearrowright X}$, arising from groups Γ and their measure preserving actions on probability spaces, $\Gamma \curvearrowright X$. Rigidity in this context occurs whenever properties of the "building data" $\Gamma, \Gamma \curvearrowright X$ are "remembered" by the isomorphism class of the resulting II_1-factor, respectively orbit equivalence (OE) relation. Such phenomena, whose study was initiated by Murray and von Neumann during 1936-1943 (see eg. [MvN36]), are quite hard to establish.

For many years progress has been slow in this direction, with much of the first period concentrating on clarifying the amenable case. This effort culminated with the work of A. Connes [Co76], showing that in fact there is no rigidity in this case: all II_1-factors of the form $L(\Gamma), L^\infty(X) \rtimes \Gamma$ with Γ amenable are isomorphic to the (unique) approximately finite dimensional II_1-factor R (also called the hyperfinite II_1-factor). Also, all free ergodic measure preserving actions of all infinite amenable groups are orbit equivalent (D. Ornstein-B. Weiss [OW80]). Similarly, it was shown that factors $L(\Gamma)$, $L^\infty(X) \rtimes \Gamma$ and equivalence relations $\mathcal{R}_\Gamma$ involving non-amenable groups Γ are not hyperfinite. But beyond this dichotomy, there has been little progress in understanding the non-amenable case, despite several important results scattered over the years, due to A. Connes (1975-1985, see e.g. [Co80B]), as well as I.M. Singer and H. Dye (1955-1963, see e.g. [Dye59]), J. Schwartz [Sch63], D. McDuff (1970), J. Feldman-C.C. Moore [FM77], D. Ornstein-B. Weiss [OW80], R. Zimmer (1977-1984, see e.g. [Zi77]), V.F.R. Jones (1980-1985, see e.g. [Jo83]), U. Haagerup (1978-1987, see e.g. [Ha85]), V. Golodets, S. Popa (1985-1986, see e.g. [Po86]), D. Voiculescu (1988-1994).

But this changed dramatically since 1999, with an explosion of rigidity results in this area, which since then has become one of the most dynamic and competitive these days. On the orbit equivalence side, this was due to breakthrough work by A. Furman (1999, based on R. Zimmer's *cocycle superrigidity*), D. Gaboriau (1999-2002, using *cost* and ℓ^2*-invariants*, see [Ga02]), N. Monod -Y. Shalom (2002, using *bounded cohomology*). In von Neumann algebras, the turning point came with deformation/rigidity theory initiated by S. Popa during 2001-2004 to study rigidity phenomena in von Neumann algebra framework, which in fact led to new progress in orbit equivalence theory as well, see [Po06].

Since entering the scene in 2005, Stefaan Vaes was involved in proving a long series of spectacular rigidity results and the solution to several long standing problems, in both von Neumann algebras and orbit equivalence theory. He has done this by vastly developing and improving all existing techniques in these areas, notably deformation/rigidity theory. We describe below the most important of this work, including the solution to two problems originating in Murray and von Neumann's 1936-1943 papers.

4.2 OE-superrigidity and $W*$-superrigidity for free, ergodic actions

The most "extreme" rigidity phenomena for equivalence relations $\mathcal{R}_\Gamma$ and group measure space II_1-factors $L^\infty(X) \rtimes \Gamma$ occur when the isomorphism class of these objects completely "remembers" the isomorphism (or conjugacy) class of the action $\Gamma \curvearrowright X$. This type of phenomena is labeled "superrigidity": a free ergodic probability measure preserving (pmp) action $\Gamma \curvearrowright X$ is *OE-superrigid* if any orbit equivalence between $\Gamma \curvearrowright X$ and another free ergodic pmp action $\Lambda \curvearrowright Y$ comes from a conjugacy of the actions; it is called W^**-superrigid* if any isomorphism between $L^\infty(X) \rtimes \Gamma$ and any other group measure space factor $L^\infty(Y) \rtimes \Lambda$ comes from a conjugacy of the actions.

One should point out that an orbit equivalence of actions implements an isomorphism of the corresponding II_1-factors (I.M. Singer 1955), in fact an OE of $\Gamma \curvearrowright X$, $\Lambda \curvearrowright Y$ corresponds exactly to an isomorphism $L^\infty(X) \rtimes \Gamma \simeq L^\infty(Y) \rtimes \Lambda$ carrying the *Cartan subalgebras* $L^\infty(X), L^\infty(Y)$ onto each other (Feldman-Moore 1977). But the converse is not true in general (A. Connes-V.F.R. Jones [CJ85]).

Thus, W^*-superrigidity of a group action $\Gamma \curvearrowright X$ amounts to proving its OE-superrigidity together with uniqueness of the Cartan subalgebra $L^\infty(X) \subset L^\infty(X) \rtimes \Gamma$, making such results very hard to establish. Indeed, while OE-superrigid actions were detected by Furman in 1999 ($SL_n(\mathbb{Z}) \curvearrowright \mathbb{T}^n$ for $n \geq 3$), Popa in 2005 (Bernoulli actions of property (T) groups), Kida in 2007 (actions of mapping class groups), no W^*-superrigid action was known to exist until Popa and Vaes discovered a whole class in their paper [PV10] (published in Inventiones).

More precisely, they introduce the class $\mathcal{G}$ of amalgamated free products $G = \Gamma_1 \star_\Sigma \Gamma_2$, where Γ_1 contains either a non-amenable subgroup with the relative property (T) or two commuting non-amenable subgroups, Σ is amenable and strictly contained in Γ_2, and there exists $g_1, ..., g_k \in G$ such that $\cap_{i=1}^k g_i^{-1} \Sigma g_i = \{1\}$, and they prove:

Theorem 1 *[PV10] If $G \in \mathcal{G}$, for every ergodic, pmp G-action on (X, μ),* *the crossed product $L^\infty(X,\mu) \rtimes G$ has a unique group measure space Cartan subalgebra, up to conjugacy.*

From this they are able to provide several classes W^*-superrigid actions, one of which is as follows:

Corollary 1 *[PV10] Let T_n be the subgroup of upper triangular matrices in $SL_n(\mathbb{Z})$ ($n \geq 3$). Then any mixing action of $SL_n(\mathbb{Z}) \star_{T_n} SL_n(\mathbb{Z})$ is W^*-superrigid.*

This breakthrough paper introduced a series of important new ideas and techniques, including a "transfer of rigidity" technique which was a key tool in several important subsequent developments by Ioana, Peterson, Houdayer. It was also crucial for the work of Ioana, Popa and Vaes that we describe below.

4.3 Superrigidity for group factors

As already mentioned, it follows from a result of Connes [Co76] that, for every amenable infinite conjugacy class (i.c.c.) group G, the group factor LG is isomorphic to the hyperfinite II_1-factor; so passing from G to LG erases all information about G (except amenability). By way of contrast, Connes [Co80A] made in 1980 the daring conjecture that, for countable i.c.c. groups with Kazhdan's property (T), the group factor determines the group. So far, this conjecture has not been checked in a single case! But it aroused much interest, in particular for the wider question: are there countable groups G for which LG determines G? In a work of extraordinary depth and technical difficulty, A. Ioana, S. Popa and S. Vaes obtained the positive answer to the latter question (see [IPV13], published in *Annals*), by showing that a certain natural "wreath-product augmentation" of any nonamenable group Γ is superrigid in this sense:

Theorem 2 *Let Γ be any countable, non-amenable group; form the wreath product $H = \Gamma \wr \mathbb{Z}$, its homogeneous space $I = H/\mathbb{Z}$, and the restricted wreath product $G = (\mathbb{Z}/2\mathbb{Z})^{(I)} \rtimes H$. Then LG determines G, more precisely if Λ is any other group such that $LG = L\Lambda$, then there exists a unitary element in LG that conjugates the elements of Λ onto scalar multiples of elements in G.*

Here LG is canonically isomorphic to $L^\infty(X) \rtimes H$ where X is the Cantor set $(\mathbb{Z}/2\mathbb{Z})^I$, which links that theorem to W^*-superrigidity for group actions. One of the key ingredients in the proof of this result is a criterion for conjugacy of subgroups of LG, showing that if $LG = L\Lambda$ and the Hausdorff-$\|.\|_2$

distance between $\mathbb{T}G, \mathbb{T}\Lambda$ is less than $\sqrt{2}$, then $\mathbb{T}G, \mathbb{T}\Lambda$ are conjugate by a unitary element in LG. The subtle proof of this criterion relies heavily on techniques that Vaes has developed in his work on quantum groups.

4.4 Rigidity for free group measure space factors

Arguably the most famous problem in operator algebra, and perhaps in all functional analysis, is whether the II1 factors $L(\mathbb{F}_n)$ of the free groups with n generators are non-isomorphic for different n's; this is known as the *free group factor problem*. A closely related, important problem, known as the *free group measure space factor problem*, is whether the group measure space factors $L^\infty(X) \rtimes \mathbb{F}_n$ are non-isomorphic for different n's, for any free ergodic measure preserving action $\mathbb{F}_n \curvearrowright X$. Both problems go back to the work of Murray and von Neumann (1936-1943) and H. Dye (1959-1963, see e.g. [Dye59]). Ever since then, they attracted a huge amount of interest. But it was only during the last 25 years that progress started to happen in this direction. For the first of these problems, this came with the advent of *free probability theory*, developed by Voiculescu and others during 1985-2005 to study free group factors, leading for instance to the striking result that the amplification of $L(\mathbb{F}_n)$ by $k = \sqrt{\frac{m-1}{n-1}}$, is isomorphic to $L(\mathbb{F}_m)$, but failing short of proving the non-isomorphism. The first progress to the second problem came with the work of Gaboriau [Ga02] on cost and ℓ^2-Betti numbers for equivalence relations (1998-2001), showing in particular that the orbit equivalence of actions $\mathbb{F}_n \curvearrowright X$ and $\mathbb{F}_m \curvearrowright Y$, implies $n = m$.

In a major recent breakthrough ([PV14], published in *Acta*), Vaes in joint work with Popa have completely settled the free group measure space factor problem, by showing that given ANY free ergodic measure preserving actions $\mathbb{F}_n \curvearrowright X$ and $\mathbb{F}_m \curvearrowright Y$, an isomorphism of the associated factors $L^\infty(X) \rtimes \mathbb{F}_n \simeq L^\infty(Y) \rtimes \mathbb{F}_m$ implies $n = m$. Moreover, they deduce the striking result that for the wreath product groups $\mathbb{Z} \wr \mathbb{F}_n = \mathbb{Z}^{(\mathbb{F}_n)} \rtimes \mathbb{F}_n$ an isomorphism $L(\mathbb{Z} \wr \mathbb{F}_n)^t \simeq L(\mathbb{Z} \wr \mathbb{F}_m)^s$ holds if and only if $\frac{n-1}{t} = \frac{m-1}{s}$. They show these results by proving that any free ergodic action of a free group, $\mathbb{F}_n \curvearrowright X$, gives rise to a II_1-factor $L^\infty(X) \rtimes \mathbb{F}_n$ with unique Cartan decomposition, a result that reduces the problem to results of D. Gaboriau and L. Bowen. The proof involves several completely novel ideas and sophisticated techniques in deformation/rigidity theory.

4.5 Fundamental groups of II_1-factors

In joint paper with Popa ([PV10B], published in Journal of the AMS), Vaes gave the first examples of (separable) II_1-factors with uncountable fundamental group, not equal to $\mathbb{R}^+$. This was open since Murray-von Neumann's 1943 paper, and it was in fact believed that only countable groups and $\mathbb{R}^+$ can appear. The examples are group measure space factors coming from free ergodic pmp actions of the free group $\mathbb{F}_\infty$. In fact, the result provides a class $\mathcal{S}$ of subgroups of $\mathbb{R}^+$ containing all countable subgroups, $\mathbb{R}^+$ itself, and subgroups of any Hausdorf dimension (after taking log), such that: any group in $\mathbb{S}$ can be realized as fundamental group of a II_1-factor and equivalence relation arising from a free ergodic pmp action of $\mathbb{F}_\infty$. This is in striking contrast with the case of actions of $\mathbb{F}_n$ with n finite, where by the result of Popa and Vaes explained above, any group measure space factor $L^\infty(X) \rtimes \mathbb{F}_n$ must have trivial fundamental group!

One should mention that the examples of $\mathbb{F}_\infty$-actions with these properties can be taken so that the associated equivalence relations have trivial outer automorphism group. The problem of the existence of actions of free groups with the equivalence relation having no outer automorphisms was open for some time. In a related paper [Va08], Vaes has constructed a class of separable II_1-factors with no non-trivial bimodules of finite Jones index, thus answering a question of Connes, while in [Va12] he settled a well known problem raised by Effros in 1975.

References

[BHV08] B. Bekka, P. de la Harpe and A. Valette, *Kazhdan's property (T).* New Mathematical Monographs 11, Cambridge University Press, Cambridge, 2008.

[Co76] A. Connes, *Classification of injective factors.* Ann. of Math. (2) 104 (1976), 73-115.

[Co80A] A. Connes, *Classification des facteurs.* In Operator algebras and applications, Part 2 (Kingston, 1980), Proc. Sympos. Pure Math. 38, Amer. Math. Soc., Providence, 1982, pp. 43-109.

[Co80B] A. Connes, *A factor of type II1 with countable fundamental group* J. Operator Theory 4 (1980), 151-153.

[CFW81] A. Connes, J. Feldman and B. Weiss *An amenable equivalence relation is generated by a single transformation,* Ergodic Theory Dynam. Systems 1 (1981), p. 431-450.

[CJ85] A. Connes and V.F.R. Jones, *Property (T) for von Neumann algebras,* Bull. London Math. Soc. 17 (1985), p. 57-62.

[Dye59] H.A. Dye *On groups of measure preserving transformation , I* Amer.J.Math. 81 (1959), p. 119-159.

[FM77] J. Feldman and C.C. Moore, *Ergodic equivalence relations, cohomology, and von Neumann algebras, II* Trans. Amer. Math. Soc. 234 (1977), p. 325-359.

[Ga02] D. Gaboriau, *Invariants ℓ^2 de relations d'équivalence et de groupes,* Publ. Math. Inst. Hautes Etudes Sci. 95 (2002), p. 93-150.

[Ha85] U. Haagerup, *A new proof of the equivalence of injectivity and hyperfiniteness for factors on a separable Hilbert space* J. Functional Anal. 62 (1985), 160-201.

[IPV13] A. Ioana, S. Popa and S. Vaes *A class of superrigid group von Neumann algebras*, Annals of Mathematics 178 (2013), 231-286.

[Jo83] V.F.R. Jones, *Index for subfactors* Invent. Math. 72 (1983), 1-25

[MvN36] F.J. Murray and J. von Neumann *On rings of operators* Ann. of Math. (2) 37 (1936), p. 116-229.

[OW80] D.S.Ornstein and B.Weiss *Ergodic theory of amenable group actions,* Bull. Amer. Math. Soc. (N.S.) 2 (1980), p. 161-164.

[Po86] S. Popa, *Correspondences*, INCREST preprint 1986.

[Po06] S. Popa, *On a class of type II_1-factors with Betti numbers invariants,* Ann. of Math. 163 (2006), 809-899.

[PV10] S. Popa and S. Vaes, *Group measure space decomposition of II_1-factors and W^*-superrigidity.* Inventiones Mathematicae 182 (2010), 371–417

[PV10B] S. Popa and S. Vaes, *Actions of $\mathbb{F}_\infty$ whose II_1-factors and orbit equivalence relations have prescribed fundamental group*, Journal of the American Mathematical Society 23 (2010), 383–403.

[PV14] S. Popa and S. Vaes, *Unique Cartan decomposition for II_1-factors arising from arbitrary actions of free groups* Acta Mathematica 212 (2014), 141-198.

[Sch63] J. Schwartz, *Two finite, non-hyperfinite, non-isomorphic factors* Comm. Pure Appl. Math. 16 (1963), 19-26.

[Va05] S. Vaes, *Strictly outer actions of groups and quantum groups.* Journal für die reine und angewandte Mathematik 578 (2005), 147-184.

[Va08] S. Vaes, *Explicit computations of all finite index bimodules for a family of II_1-factors.* Ann. Sci. Ec. Norm. Sup. 41 (2008), 743-788.

[Va12] S. Vaes, *An inner amenable group whose von Neumann algebra does not have property Gamma.* Acta Math 208 (2012), 389-394.

[Zi77] R.J. Zimmer, *Hyperfinite factors and amenable ergodic actions* Invent. Math. 41 (1977), 23-31.

Alain Connes
Collège de France
alain@connes.org

Vaughan Jones
Vanderbilt University
vaughan.f.jones@Vanderbilt.Edu

Sorin Popa
UCLA
popa@math.ucla.edu

Alain Valette
Université de Neuchâtel
alain.valette@unine.ch

Koninklijke Vlaamse Academie van België voor Wetenschappen en Kunsten
The Royal Flemish Academy of Belgium for Science and The Arts

Academy Contact Forum
“Coding Theory and Cryptography VI”
Friday, October 2, 2015, Brussels, Belgium

The contact forum is organized by The Royal Flemish Academy of Belgium for Science and The Arts, and the research groups Incidence Geometry (Ghent University), COSIC (K.U.Leuven), and ETRO (VUB).

It will be held in Brussels at the Paleis der Academiën, Hertogsstraat 1, B-1000 Brussel.

List of invited speakers:

- Elena Andreeva (K.U.Leuven, Belgium)
- Daniele Bartoli (Ghent University, Belgium)
- Wouter Castryck (Ghent University, Belgium)
- Tanja Lange (Eindhoven University of Technology, The Netherlands)
- Fernando Pérez-González (University of Vigo, Spain)
- Kai-Uwe Schmidt (Otto-von-Guericke Universität Magdeburg, Germany)
- Fréderik Vercauteren (K.U.Leuven, Belgium)

There is no registration fee. Please register before September 26, 2015, by sending an e-mail to **bcrypt@cage.ugent.be** with your name and affiliation. All information will be made available on `http://cage.ugent.be/~ls/website2015/contactforum2015.html`

The organizers:

A. Dooms (VUB)
S. Nikova (K.U.Leuven)
B. Preneel (K.U.Leuven)
V. Rijmen (K.U.Leuven)
L. Storme (UGent)

This contact forum is sponsored by: The Royal Flemish Academy of Belgium for Science and The Arts, research groups Incidence Geometry (Ghent University), COSIC (K.U.Leuven), ETRO (VUB), BCRYPT: Belgian Fundamental Research on Cryptology and Information Security, and the Scientific Research Network “Veilige ICT”.

Wandering colloquium
Getaltheorie in het vlakke land
Arithmétique en plat pays

Monday October 19, 2015, Ghent, Belgium

In 2010, the tradition was started to regularly organize colloquia on number theory in the north of France and Flandres (`http://math.univ-lille1.fr/~bhowmik/seminaire/ALL_2010.html`).
On October 19, 2015, the colloquium returns to Ghent, Belgium.
The colloquium is organized by the research groups Algebra and Incidence Geometry (Ghent University), COSIC (K.U.Leuven), and Département de Mathématiques (Université de Lille I).

List of invited speakers:

- Jennifer Balakrishnan (University of Oxford, England)
- Jan De Beule (Ghent University, Belgium)
- Steven Galbraith (University of Auckland, New Zealand)
- Florent Jouve (Université Paris-Sud, France)
- Martin Orr (Imperial College London, England)

The lectures take place in Auditorium Emmy Noether, Building S25, Department of Mathematics, Ghent University, Krijgslaan 281, B-9000 Ghent.
The location of Auditorium Emmy Noether can be found on the website `http://cage.ugent.be/zwc/map.en.html`. The auditorium Emmy Noether (building S25) is the red building with number 40.25 on this website.

There is no registration fee. All information will be made available via the website `http://math.univ-lille1.fr/~bhowmik/seminaire/APP_oct_2015.html`. Participants are requested to register before October 12, 2015, via this website.

This colloquium is sponsored by: research groups Incidence Geometry (Ghent University), COSIC (K.U.Leuven), Département de Mathématiques (Université de Lille I), and the Scientific Research Network "Veilige ICT".

The organizers:

G. Bhowmik (Université de Lille I)
W. Castryck (UGent)
L. Storme (UGent)
F. Vercauteren (K.U.Leuven)

Faculté des Sciences

UMONS
Université de Mons

Françoise BASTIN
Catherine FINET
Karl GROSSE-ERDMANN

Groupe de Contact
FNRS Group – Functional Analysis

Une journée autour d'une même famille: Analyse et Probabilités

10h00 Gilles Godefroy (Université Pierre et Marie Curie, Paris)
Espaces de Banach libres

11h15 Benjamin Arras (Université de Liège)
Autour du théorème centrale limite

14h30 Jochen Wengenroth (Universität Trier)
Dérivations sur des ensembles petits

15h45 Laurent Simons (HELMo Gramme, Liège)
La bijection de Cantor

Jeudi 5 Novembre 2015
Le Pentagone – Salle 0A11/rez-de-chaussée
Avenue du Champ de Mars, 6
7000 Mons

Invitation cordiale à tous

Institut des Hautes Etudes de Belgique

CONFERENCE

JEUDI 22 OCTOBRE 2015 à 19 heures

Université Libre de Bruxelles, Bâtiment S, Rez-de-chaussée

Avenue Jeanne, 44, 1050 Bruxelles

Plonger des graphes dans des espaces vectoriels euclidiens... ou pas euclidiens!

Alain Valette,

Professeur à l'université de Neuchâtel.

Pour comprendre de gros graphes, il peut être intéressant de les plonger dans un espace euclidien de grande dimension: cela se fait par exemple en bioinformatique.

Mais on peut aussi considérer d'autres distances que la distance euclidienne, par exemple la distance L^1 (ou de Manhattan).

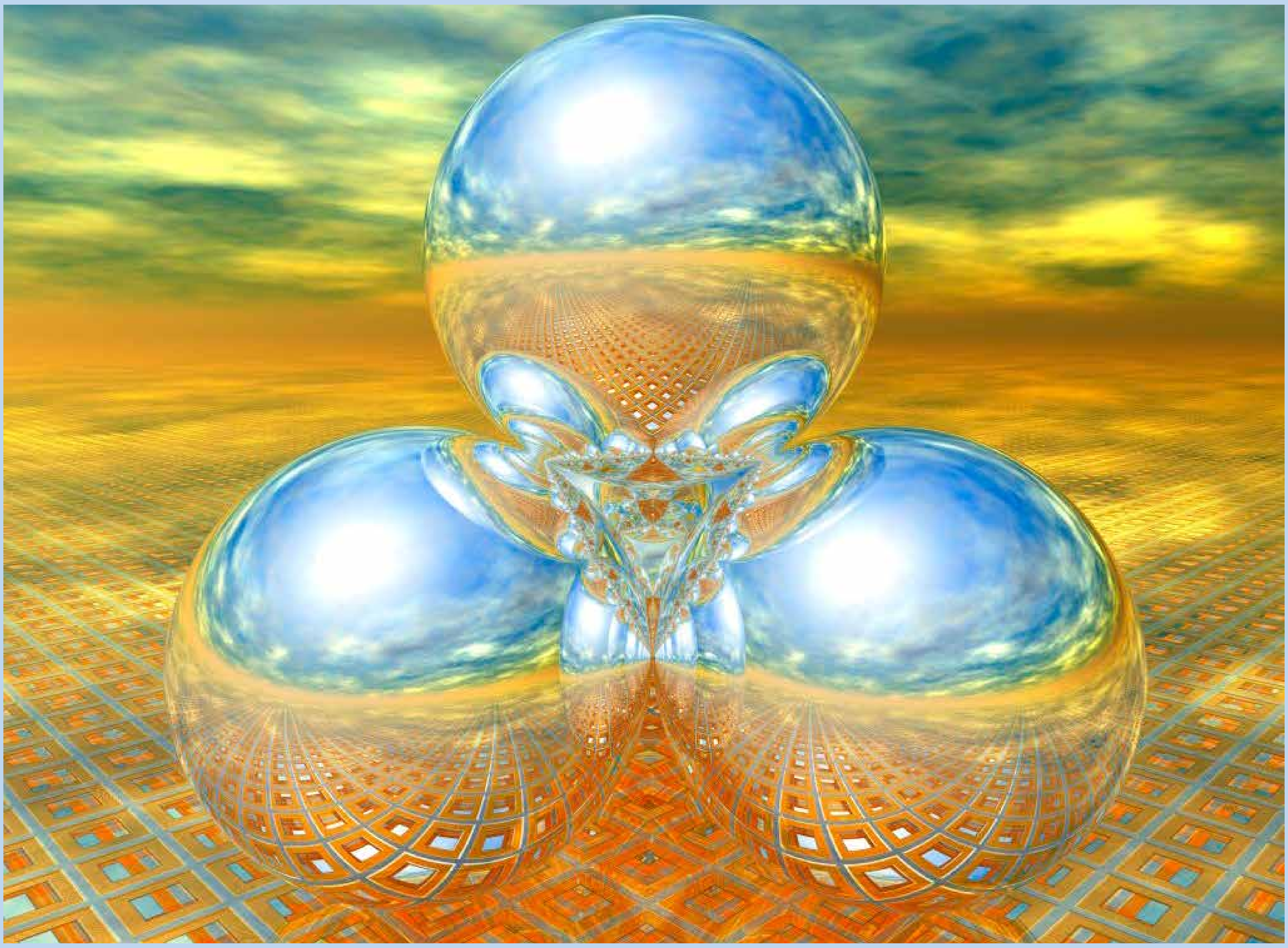
La distorsion d'un graphe, introduite en 1985 par le mathématicien belge Jean Bourgain, mesure la difficulté à le plonger dans un espace L^1.

Ces travaux de Bourgain ont trouvé des applications en informatique théorique, par exemple dans le problème SPARSEST CUT, qui demande de trouver le nombre minimal d'arêtes à retirer à un graphe pour le déconnecter.

Entrée libre

IMAGINARY

wiskunde in sprankelende beelden



Gent, Campus Sterre, Gebouw S30,
28 september t.e.m. 23 oktober 2015

Leuven, Centrale Bibliotheek,
7 november t.e.m. 28 november 2015

Kortrijk, Kulak, 4 januari t.e.m. 22 januari 2016

Antwerpen, Campus Middelheim,
1 februari t.e.m. 19 februari 2016

Diepenbeek, Agoralaan, gebouw D,
29 februari t.e.m. 18 maart 2016

Brussel, Pleinlaan 2, 11 april t.e.m. 29 april 2016

Een reizende wiskundetentoonstelling, voor het eerst in België.
De schitterende beelden, unieke 3D-objecten en interactieve visualisaties laten niemand onberoerd.

GRATIS TE BEZOEKEN.

Volg ons aanbod voor leraren, scholen en gratis geleide bezoeken op **www.imaginarymaths.be**

EEN INITIATIEF VAN VLAAMSE WISKUNDE OLYMPIADE I.S.M. DE VLAAMSE UNIVERSITEITEN

V.U. Paul Igodt voor Vlaamse Wiskunde Olympiade vzw - Foto: Luc Benard

ORGANISATOREN

VWO
KU LEUVEN
UNIVERSITEIT GENT
universiteit hasselt KNOWLEDGE IN ACTION
Universiteit Antwerpen
Vrije Universiteit Brussel
KU LEUVEN kulak

PARTNERS

SBG
Materialise innovators you can count on
die Keure
RICHTING MORGEN MET WETENSCHAP, TECHNOLOGIE, CREATIVITEIT & INNOVATIE
Vlaamse overheid
didakta bij ons zit u goed
Klaus Tschira Stiftung gemeinnützige GmbH
KTS
Mathematisches Forschungsinstitut Oberwolfach

IMAGINARY

wiskunde in sprankelende beelden

Citrus $x^2+z^2=y^3(1-y)^3$

Gent, Campus Sterre, Gebouw S30, 28 september t.e.m. 23 oktober 2015

Leuven, Centrale Bibliotheek, 7 november t.e.m. 28 november 2015

Kortrijk, Kulak, 4 januari t.e.m. 22 januari 2016

Antwerpen, Campus Middelheim, 1 februari t.e.m. 19 februari 2016

Diepenbeek, Agoralaan, gebouw D, 29 februari t.e.m. 18 maart 2016

Brussel, Pleinlaan 2, 11 april t.e.m. 29 april 2016

Een reizende wiskundetentoonstelling, voor het eerst in België.
De schitterende beelden, unieke 3D-objecten en interactieve visualisaties laten niemand onberoerd.

GRATIS TE BEZOEKEN.

Volg ons aanbod voor leraren, scholen en gratis geleide bezoeken op **www.imaginarymaths.be**

EEN INITIATIEF VAN VLAAMSE WISKUNDE OLYMPIADE I.S.M. DE VLAAMSE UNIVERSITEITEN

ORGANISATOREN

VWO
KU LEUVEN
UNIVERSITEIT GENT
universiteit hasselt KNOWLEDGE IN ACTION
Universiteit Antwerpen
Vrije Universiteit Brussel
KU LEUVEN kulak

PARTNERS

SBG
Materialise innovators you can count on
die Keure
RICHTING MORGEN MET WETENSCHAP, TECHNOLOGIE, CREATIVITEIT & INNOVATIE
Vlaamse overheid
didakta bij ons zit u goed
Klaus Tschira Stiftung gemeinnützige GmbH
KTS
Mathematisches Forschungsinstitut Oberwolfach

V.U. Paul Igodt voor Vlaamse Wiskunde Olympiade vzw - Visualisatie: Herwig Hauser