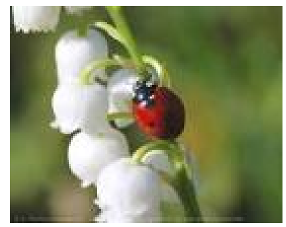
Newsletter

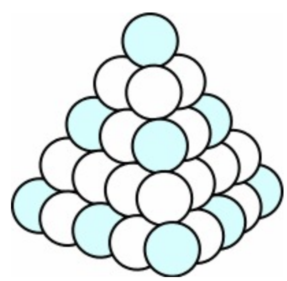
BELGIAN MATHEMATICAL SOCIETY

108, May 15, 2016

Comité National de Mathématique CNM

NCW Nationaal Comité voor Wiskunde





Newsletter of the Belgian Mathematical Society and the National Committee for Mathematics

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Contents

Welcome to the May edition of the BMS-NCM Newsletter!

The BeNeLux Mathematical Congress was held in Amsterdam on March 22–23. There were more than 250 participants and about 30 of them were from Belgium. The conference was very well-organised by our colleagues of the Royal Dutch Mathematical Society (KWG). Besides the first N.G. de Bruijn prize, the KWG prize for PhD students was also awarded during this congress. The De Bruijn prize went to Prof. Marius Crainic (Utrecht) and the PhD prize was awarded to a Belgian participant: Geoffrey Janssens from the Vrije Universiteit Brussel. You can read more about the prize and Geoffrey's PhD research further in this letter. Congratulations to Geoffrey!

Our previous Newsletter was published the day after π-day. At that time I was not aware that π-day 2016 was a very special one. It was precisely on π-day that Maryna Viazovska decided to post a paper on ArXiv in which she proves that no packing of unit balls in Euclidean space $\mathbb{R}^8$ has density greater than that of the E_8-lattice packing. This is a high dimensional version of *Kepler's Conjecture*.

In 1611 Johannes Kepler stated that the most efficient way to stack equal-sized spheres in space is the familiar pyramidal piling of oranges seen in grocery stores. Despite the problem's seeming simplicity, it was not settled until 1998, when Thomas Hales (Pittsburgh) finally proved Kepler's conjecture in 250 pages of mathematical arguments combined with mammoth computer calculations.

Finding the best packing of equal-sized spheres in a high-dimensional space should be even more complicated than the three-dimensional case Hales solved, since each added dimension means more possible packings to consider. Yet mathematicians have long known that two dimensions are special: in dimensions 8 and 24, there exist dazzlingly symmetric sphere packings called E_8 and the *Leech lattice*, respectively, that pack spheres better than the best candidates known in other dimensions.

Maryna Viazovska, a postdoctoral researcher at the Berlin Mathematical School and the Humboldt University of Berlin, has come up with a proof of Kepler's Conjecture in dimension 8. Her work uses the theory of modular forms, powerful mathematical functions that, when they can be brought to bear upon a problem, seem to unlock huge amounts of information. In this case, finding the right modular form allowed Viazovska to prove, in a mere 23 pages, that E_8 is the best eight-dimensional packing.

It's possible to build an analogue of the pyramidal orange stacking in every dimension, but as the dimensions get higher, the gaps between the high-dimensional oranges grow. By dimension eight, these gaps are large enough to hold new oranges, and in this dimension only, the added oranges lock tightly into place. The resulting eight-dimensional sphere packing, known as E_8, has numerous applications in many other parts of mathematics and physics.

The Leech lattice is similarly constructed by adding spheres to a less dense packing, and it was discovered almost as an afterthought. In the 1960s, the British mathematician John Leech was studying a 24-dimensional packing that can be constructed from the *Golay code*, an error-correcting code that was later used to transmit the historic photos of Jupiter and Saturn taken by the Voyager probes. Shortly after Leech's article about this packing went to press, he noticed that there was room to fit additional spheres into the holes in the packing, and that doing so would double the packing density.

Within a week, Viazovska, along with Cohn and three other mathematicians, successfully extended her method to cover the Leech lattice in dimension 24 too. Their paper is also on ArXiv.

It is nice to see progress on a problem as old as Kepler's Conjecture and at the same time get another confirmation of the beauty and very deep importance of both the E_8-lattice and the Leech lattice.

Philippe Cara,
president of the BMS

1 Meetings, Conferences, Lectures

1.1 May 2016

Chaire De La Vallée Poussin

Arithmetic of Algebraic Groups

Professor Raman PARIMALA (Atlanta)

May 17-19, 2016, UCL

Program

- Tuesday May 17, 2016 – 16:30: Inaugural Lecture (followed by a reception) *Arithmetic properties of function fields of p-adic curves*
- Wednesday May 18, 2016 – 10:30 : Lecture 2 *Quadratic forms and Brauer groups I*
- Wednesday May 18, 2016 – 15:30 : Lecture 3 *Quadratic forms and Brauer groups II*
- Thursday May 19, 2016 – 15:30: Lecture 4 *Reciprocity obstructions to Hasse principle*

See more information at the address https://www.uclouvain.be/684817.html

Maths Jam: fun, games and creative maths

Antwerp, May 24, 2016

On May 24th at 19h in Agora Caffee, Grote Kauwenberg 2, Antwerpen. See announcement at the end of this newsletter.

Brauer groups, Hopf algebras and monoidal categories

A conference in honour of Stef Caenepeel on the occasion of his 60th birthday

May 24-27, 2016, Turin, Italy

In 2016, Stef Caenepeel (former president of the BMS) will turn 60. To celebrate this joyful event, a conference is organized in Turin, Italy, from Tue 24 - Fri 27 May 2016. Hereby, you are cordially invited to participate in this meeting.

The conference website can be found on the address:

http://homepages.vub.ac.be/~hopfalgb

If you are interested to attend the conference, please fill out the registration form that is available on this site. Feel free to forward this message to everyone who might be interested.

We hope to be welcoming you in Turin!!

Best wishes,
The organizers,
Ana Agore, Alessandro Ardizzoni, Sorin Dascalescu,
Isar Goyvaers, Gigel Militaru, Joost Vercruysse

1.2 June 2016

Second joint Conference of the

Belgian, Royal Spanish and Luxembourg Mathematical Societies

June 6–8, 2016, Logroño, Spain

Second announcement

- WEBSITE : http://bsl.unirioja.es
- CONFERENCE SITE

  The conference will take place at convention centre Riojaforum and at Faculty of Science and Technology in Logroño, Spain
- REGISTRATION

  The registration is open now until May 31, 2016. The registration fee is 60 EUR. PhD students have a 40 EUR registration fee. More information on https://bsl.unirioja.es/registration.php

  Information on special arrangement for accommodation available on https://bsl.unirioja.es/hotels.php Special prices are guaranteed until April 25, 2016
- PROGRAMME

  Besides ten special sessions and posters sessions, there will be seven plenary talks. More information on the website.
- PLENARY SPEAKERS
  - *Sara Arias de Reyna*
    Faculté des Sciences, de la Technologie et de la Communication Université du Luxembourg.
  - *María Jesús Carro*
    Departamento de Matemática Aplicada y Análisis Facultad de Matemáticas Universidad de Barcelona. (Valdivia Lecture)
  - *Raf Cluckers*
    Université Lille 1 Sciences et Technologies Laboratoire Painlevé.
  - *Sergei Merkulov*
    Faculté des Sciences, de la Technologie et de la Communication Université du Luxembourg.
  - *Johannes Nicaise*
    Imperial College London Department of Mathematics.

- *Jesús María Sanz Serna*
  Departamento de Matemáticas Universidad Carlos III
- *Anton Thalmaier*
  Unité de Recherche en Mathématiques, FSTC Université du Luxembourg.

- SEE POSTER at the end of this Newsletter

1.3 July 2016

Workshop on Computability Theory 2016

Ghent, July 4–5, 2016

See the announcement and more information at the address

http://wct.math.uconn.edu/wctghent/index.html

1.4 August 2016

Brussels Summer School in Mathematics

Brussels, August 1–5, 2016

The Brussels Summer School of Mathematics is organised every year during the first week of August at the Université libre de Bruxelles. It consists in a full week of courses on a wide range of mathematical topics (algebra, calculus, geometry, differential geometry, logic, probability, statistics, topology, mathematical physics, ...). Courses are taught in French or English, and the speakers are researchers (phd, postdoc or professor) working in a Belgian university or from abroad.

The courses are intended to be accessible to any person with a basic knowledge of mathematics. The purpose of the BSSM is not to provide a formal introduction to any particular theory but rather to give the audience a chance to glimpse the workings, beauty and importance of cutting-edge mathematical research. In order to achieve this goal the emphasis of most talks is put on important or intriguing problems which allow for short and enthralling solutions.

The BSSM is not intended for specialists nor is it a vulgarization exercise. The aim of this summer school is to bridge the gap between these two important modes of scientific expression, by inviting a non-specialized audience (of students, teachers, professionals and amateurs) to discover the joys and beauty of advanced mathematical research.

Registration and programme on http://bssm.ulb.ac.be

During this BSSM, on August 3rd, **Jean Van Schaftingen (UCL)** will also deliver the **BMS Godeaux Lecture 2016**.

These lectures honoring the memory of Lucien Godeaux are organized with the assets of the Belgian Center for Mathematical Studies which were transferred to the BMS after the dissolution of this Center. Lucien Godeaux (1887-1975) was one of the world's most prolific mathematicians (with more than 700 papers published) and took many initiatives to encourage young mathematicians to communicate their research. He was the founder of the Belgian Center for Mathematical Studies in 1949.

1.5 September 2016

The Beautiful Impact of Mathematics in Society (BIMS)

Brussels, September 21st, 2016

The Vrije Universiteit Brussel organises a one day colloquium on the broad impact of mathematics in our society. Part of this event will be in Dutch.

De Vrije Universiteit Brussel nodigt u van harte uit om de conferentie "The Beautiful Impact of Mathematics in Society" bij te wonen op de VUB-campus in Etterbeek op 21 september 2016. Wiskunde is een onmisbaar fundament van alle wetenschap en technologie, toch is dit vakgebied niet altijd even zichtbaar in onze samenleving. Tijdens deze conferentie willen we wiskunde als zelfstandige discipline in de spotlight plaatsen en aan een geïntegreerde aanpak voor onderwijs, lerarenopleiding, onderzoek, maatschappij en innovatie werken.

We hebben hiervoor een boeiend dagprogramma samengesteld met 6 inspirerende sprekers:

- Wil Schilders (Directeur van het Platform Wiskunde Nederland)
- Raymonda Verdyck (Hoofd van het Gemeenschapsonderwijs GO!)
- Etienne Ghys (Wiskunde onderzoeker met een hart voor wetenschapscommunicatie)
- Jan Bogaerts (Statisticus aan de top van de Europese Organisatie voor het Onderzoek naar en Behandeling van Kanker)
- Ton Kalker (Wiskundige uit het bedrijfsleven met een neus voor innovatie)
- Simon Singh (Schrijver van de blockbusters "Fermat's Last Theorem" en "The Code Book")

Meer informatie op

https://www.facebook.com/events/1707616246192506/

We sluiten de dag af met een echte "Maths Jam" waar we ons tussen pot en pint buigen over allerlei wiskundige raadsels: https://www.facebook.com/events/234181823610457/

2 PhD theses

No thesis announcements this time.

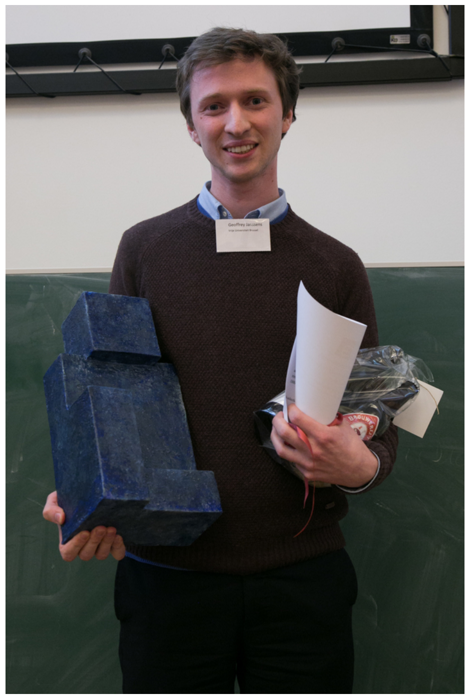
Will you defend your PhD soon? Do you have a student who is about to obtain his PhD?

Grab the opportunity to announce it in our Newsletter!

Next Newsletter will appear on September 15. The deadline for contributions is September 10. Contact Françoise Bastin <F.Bastin@ulg.ac.be> with title, abstract and defence date/place.

3 Geoffrey Janssens wins KWG PhD Prize

On March 23rd, Geoffrey Janssens (VUB) was ranked first among the 6 finalists of the KWG PhD Prize. Any PhD student or recent PhD from a Belgian, Dutch or Luxembourg university could postulate for this prize. The jury selected 6 nominees based on their curriculum and mathematical work. During the final day of the BeNeLux conference the nominees had the opportunity to present one aspect of their work. Presentations had to be aimed at a general mathematical audience, and the student's ability to make the subject accessible to non-specialists was an important criterion for winning the award.



In what follows, Geoffrey describes his research.

A glimpse into the Asymptotics behind Polynomial Identities in algebras

A classical object in mathematics is an algebra over some field F. In this text we will always assume that the scalers live in $\mathbb{C}$ (even though all results hold in all fields of characteristic 0). Remember that a set A is called an algebra over $\mathbb{C}$ if it is a vector space over $\mathbb{C}$ (i.e. one can add and do scalar multiplication), it is a (associative) ring with unit element (i.e. we can not only add but also multiply two elements from A in a compatible way which is expressed by distribution) and finally $\alpha ab = a\alpha b = ab\alpha$ for all $a, b \in A$ and $\alpha \in \mathbb{C}$.

Furthermore, *the algebras will always be finitely generated over* $\mathbb{C}$. Thus there exists a finite number of elements $a_1, \ldots, a_n$ such that $A = \mathbb{C}\langle a_1, \ldots, a_n \rangle$ (is generated as an algebra over $\mathbb{C}$ by the elements $a_1, \ldots, a_n$). For the reader not familiar with algebras, imagine $A = M_n(\mathbb{C})$ is a matrix ring or $A = \mathbb{C}[x_1, \ldots, x_n]$ a polynomial ring in commutative variables. One could try to classify all finitely generated algebras over $\mathbb{C}$ up to isomorphism but this is of course hopeless. So lets rather try to describe the possible 'rough shapes' of A. To make this more precise some definitions are needed.

Definition 1 *A non-zero polynomial $f(x_1, \ldots, x_n) \in F\langle X \rangle$, in some non-commutative indeterminates $x_1, \ldots, x_n$,* *is called a polynomial identity of A if $f(a_1,\dots,a_n)=0$ for any $(a_1,\dots,a_n)\in A^n$, denoted $f\equiv_A 0$. The set of all polynomial identities of A is denoted* $\mathrm{id}(A)=\{f\in F\langle X\rangle \mid f\equiv_A 0\}$.

Such polynomial needn't exist. However in many cases it does.

For example, by definition, an algebra A is commutative if and only if $ab-ba=0$ for all $a,b\in A$ or put otherwise if and only if $f(x,y)=xy-yx\equiv_A 0$.
Also our main example $A=M_n(\mathbb{C})$ satisfies some polynomial identity. Indeed

$$f_{n^2+1}:=\sum_{\sigma\in Sym_{n^2+1}}\mathrm{sgn}(\sigma)x_{\sigma(1)}\dots x_{\sigma(n^2+1)},$$

called the standard identity, is satisfied by $M_n(\mathbb{C})$.[1] As an important consequence *any finite dimensional algebra satisfies a PI!* All this implies that the class of PI-algebras is a large one.

We now have all the ingredients to refine our thoughts into the following

"Classify all finitely generated algebras over $\mathbb{C}$ up to PI equivalence"

where we say that A is PI-equivalent [2] to B if $\mathrm{id}(A)=\mathrm{id}(B)$.

So in the question above we do not distinguish 'small relations' and in a way we try to describe the possible rough shapes of algebras delivered by polynomial identities. Let us reformulate the problem. First fix some set $S=\{f_i\in F\langle X\rangle\}$ of polynomials and let $\mathcal{V}(S)=\{A\in \mathrm{Alg}_{\mathbb{C}}\mid S\subseteq \mathrm{id}(A)\}$, called *the variety*[3] *corresponding to S.* For example if $S=\{xy-yx\}$ then $\mathcal{V}(S)$ is the set of all abelian algebras. Now, in other words, *the goal is to describe $\mathcal{V}(S)$ for all possible sets S.*

More precisely we would like a full list of invariants distinguishing (and thus determining) all varieties. Unfortunately, this is (yet) completely out of reach. Remember that an invariant is a number that one associates to any algebra (and variety) and which does not change under PI-equivalence[4]. For example, the area of a triangle is an invariant with respect to isometries of the Euclidean plane. Also the determinant of a matrix is one and fits perfectly in our mindset. More precisely, the determinant notices the difference between invertible and non-invertible matrices. We will now introduce two invariants which turn out to deliver much information.

Denote $P_n(\mathbb{C})=\mathrm{span}_{\mathbb{C}}\{X_{\sigma(1)}\dots X_{\sigma(n)}\mid \sigma\in S_n\}$, the set of all multilinear polynomials over $\mathbb{C}$. Then the non-negative integer $c_n(A)=\dim_{\mathbb{C}}\frac{P_n(\mathbb{C})}{P_n(\mathbb{C})\cap \mathrm{id}(A)}$ is called the *n-th codimension* of the algebra A. Thus we get a function $f:\mathbb{N}\to\mathbb{N}:n\mapsto c_n(A)$. Remark that this function depends on $\mathrm{id}(A)$ rather then A, thus it is constant on PI-equivalence classes. Regev proved in the 70's that this function is exponentially bounded, i.e. there exist some $a\in\mathbb{R}$ such that $c_n(A)\le a^n$ for all n. Moreover he conjectured a much stronger statement.

Conjecture 1 (Regev, 70's) *There exists $t\in\frac{1}{2}\mathbb{Z}, d\in\mathbb{Z}$ and $c\in\mathbb{R}$ such that $c_n(A)\simeq cn^td^n$ where $f\simeq g$ iff* $\lim_{n\to\infty}\frac{f}{g}=1$.

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[1] This roughly follows from two observations, on one hand f_{n^2+1} is multilinear thus it is sufficient to substitute basis elements of $M_n(\mathbb{C})$, which are in a total number of n^2. And on the other hand the polynomial is alternating thus if we substitute two times the same element the polynomial vanish.

[2] PI-equivalence is really an equivalence relation.

[3] This is a variety in the sense of Birkhoff. Also note that these varieties encompass the equivalence classes of the PI-equivalence relation. Indeed $A\sim_{PI}B$ iff $\mathcal{V}(\mathrm{id}(A))=\mathcal{V}(\mathrm{id}(B))$ where $\mathcal{V}(\mathrm{id}(A))=\{C\in\mathrm{Alg}_{\mathbb{C}}\mid \mathrm{id}(A)\subseteq\mathrm{id}(C)\}$.

[4] Thus some number that is constant on each equivalence class.

In a breakthrough paper [3] Giambruno and Zaicev proved in 1998 that $d = \lim_{n\to\infty} \sqrt[n]{c_n(A)} \in \mathbb{N}$. Moreover, they did not simply prove that this integer exists, but even delivered a computable algebraic formula relating 'd' to how the 'nice elements' (i.e. semisimple) and the 'bad elements' (i.e. the Jacobson radical $J(A)$) interact to each other. For example if A is abelian[5] then $d = 1$ and $d = m^2$ if $A = M_m(\mathbb{C})$. Finally Berele and Regev proved in 2008 the full conjecture, see [2].

Unfortunately no concrete information concerning t can be extracted from the proof of Berele and Regev and was therefore a main open problem in asymptotic PI theory. Recently, in October 2015, joint with Yakov Karasik and Eli Aljadeff [1], we found a concrete formula for t!

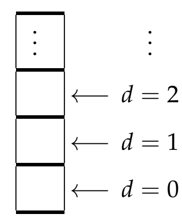
Theorem 1 *If A is a (basic) algebra, then*

$$t(A) = \frac{\textit{number of simple repr.} + \textit{sum of dimensions simple repr.}}{2} + (s-1)$$

where $s \in \mathbb{Z}_+$ smallest integer such that $J(A)^s = 0$

Now that we have these invariants containing useful algebraic info on A, it is time to use them for the problem of classifying varieties!

Start by distributing all varieties into layers according to their PI-exponent (see picture). Let S now be a fixed set of polynomials, consider $\mathcal{V}(S)$ and suppose that its exponential growth is d. It could be that by adding polynomials to S we get a strict smaller variety with a strict smaller invariant 'd'. If this always happens $\mathcal{V}(S)$ is called a *minimal variety*. These have been classified in [4] and the answer surprisingly turns out to be very elegant .



In a next step one tries to differentiate varieties in a fixed layer (i.e. we fix the invariant 'd'). Again we can distribute them into smaller layers depending on the polynomial growth, thus the invariant 't'.

Joint with Karasik we classify the varieties minimal with respect to the invariant 't' with a fixed exponential growth 'd'. Again an elegant answer pops up. Nevertheless, a long interesting road towards describing all varieties (and thus all algebras up to PI-equivalence) is still awaiting. However there is nothing as nice as a walk on a sunny day!

References

[1] E. Aljadeff, G. Janssens, Y.Karasik, The Polynomial Part of the Codimension Growth of Affine PI algebras, submitted (2015), 22 pp

[2] Berele, A., Regev, A., Asymptotic behaviour of codimensions of p. i. algebras satisfying Capelli identities, *Trans. Amer. Math. Soc.* **360** (2008), no. 10, 5155–5172..

[3] Giambruno, A., Zaicev, M.V, On codimension growth of finitely generated associative algebras, *Adv. Math.* **140** (1998), no. 2, 145–155.

[4] Giambruno, A., Zaicev, M.V, Minimal varieties of algebras of exponential growth, *Adv. Math.* **174** (2003), 310–323

We would like to publish more news from our members in the Newsletter! Did you recently win a prize or have you written a book? Let us know!

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[5] Note that in this $\frac{P_n(\mathbb{C})}{P_n(\mathbb{C})\cap \mathrm{id}(A)} = \mathrm{span}_{\mathbb{C}}\{x_1 \cdots x_n\}$.

4 Miscellaneous

4.1 From UMons

Postes vacants à l'Université de Mons-Hainaut (Belgique)

- Un poste d'assistant, temps plein, est à pourvoir dans le service de probabilités et statistique (chef de service: Karl Grosse-Erdmann), à partir du 15/09/2016.
- Un poste d'assistant, mi-temps, est à pourvoir dans le service d'analyse mathématique (chef de service: Catherine Finet), à partir du 15/09/2016.

Les adresses électroniques de contact sont: kg.grosse-erdmann@umons.ac.be et catherine.finet@umons.ac.be

4.2 From UGent

The faculty of Sciences has a vacancy for a professorship, starting from October 1st 2016. It concerns a full time position as Professor in the rank of Assistant Professor tenure track in the Department of Applied Mathematics, Computer Science and Statistics, charged with academic teaching, academic research and carrying out academic services in one of the following disciplines: artificial intelligence or big data science or mathematical modelling.

Applications should be submitted no later than May 25, 2016 at 23h59 (CET) by e-mail.

For more information about this vacancy please have a look at

https://edit.ugent.be/en/work/vacancies/professorial-staff/full-time-position-as-professor-in-the-rank-of-assistant-professor-tenure-track-in-the-discipline-of-artificial-intelligence-big-data-science-mathematical-modelling/

4.3 Meeting for Bourgain

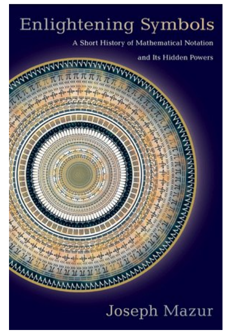
Please note that the Institute for Advanced Studies organizes (at Princeton) a **meeting in honour of Jean Bourgain next May (May 21-24)**: see at the pages https://www.math.ias.edu/bourgain16

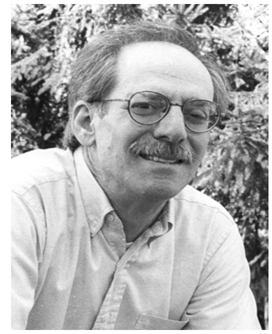
5 History, maths and art, fiction, jokes, quotations ...

5.1 Book reviews

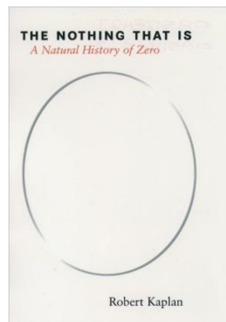
To read during long sunny (;-) summer afternoons or ... anytime!!, please find here some reviews from A. Bultheel.

Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Powers. by *Joseph Mazur.* Princeton University Press, 2014, ISBN 978-0691-1-5463-3 (hbk), 285 pp.
The Nothing that Is: A Natural History of Zero. by *Robert Kaplan.* Oxford University Press, 2000, ISBN 978-0195142372 (hbk), 240 pp.
Finding Zero. A Mathematician's Odyssey to Uncover the Origins of Numbers by *Amir D. Aczel.* Palgrave Macmillan Trade, 2015, ISBN 978-1137279842 (hbk), 256 pp.

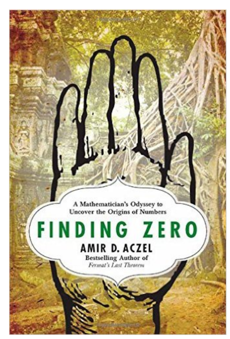


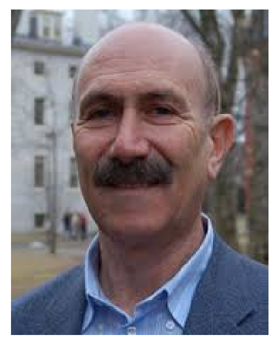


Joseph Mazur



Robert Kaplan





Amir Aczel

When you are a LaTeX user, you will know that there are several counters to number sections, definitions, footnotes etc. You can select the way in which they are represented. One possibility is to select `\arabic` to get the usual numbers. After reading these books, you know that these Arabic numbers should actually be called Hindu numerals or perhaps Hindu-Arabic because they originate from India and were brought to other societies directly such a the Khmer, and to our regions though the Arabs. *Lam Lay Yong* and *Ang Tian Se* even suggest that the system was originally Chinese[1]. The book by Aczel is all about a forgotten paper by *Georges Cœdès*[2] (pronounce *sédès*) of 1931 who discovered decimal notation, including the zero, in a Cambodian stele that was 2 centuries older than was accepted at that time. In another paper by Cœdès[3] of 1968, he explains how the Southeast Asian states were influenced by the Hindu society. Hence proving that our digits, even including the zero that had to be re-invented in the West much later, has an Hindu origin. Kaplan also discusses the origin of the different notations for numbers, but in particular the origin of zero. About this zero, there is a lot of folklore and speculation circulating. Kaplan discusses many of them, but is not very specific concerning the actual origin. Perhaps there are many possible explanations because people did really talk or discuss about there being none or nothing and there were several ways to denote a zero, but whether this was a way of indicating that there was no number or whether there was indeed a number, namely zero, will probably always be speculation.

LaTeX has also the possibility to select `\Roman` to get the familiar Roman numerals, but also you can select `\alph` for an alphabetic numbering or as sometimes used for footnotes some symbols can be selected like †, ‡, §, etc. with `\fnsymbol`. The latter system using a different symbol for every number obviously only works for small numbers. In principle, they could be used to generate a number system This is what the Roman numeral also do: only a few symbols were selected and used as numeral so compose a larger number. The Roman system uses I, V, X, L, C, D, M to denote 1, 5, 10, 50, 100, 500, 1000, which suffices to denote current years, but it would be a problem to use them in cosmology or nano-science. The Egyptians had hieroglyphics

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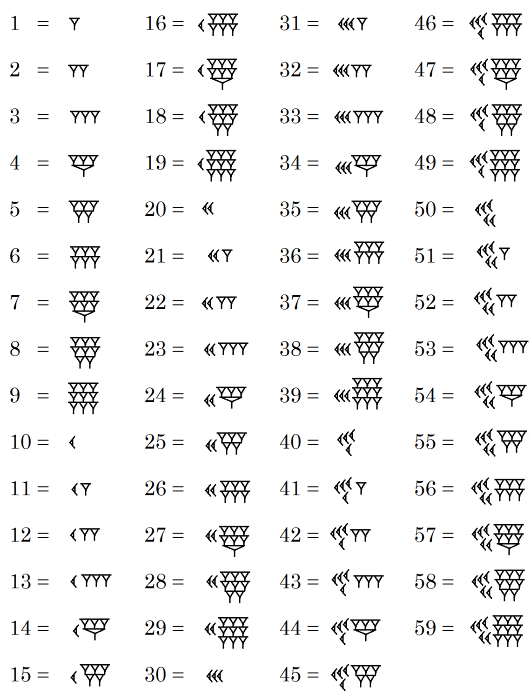
[1] Lam Lay Yong and Ang Tian Se. *Fleeting Footsteps: Tracing the Conception of Arithmetic and Algebra in Ancient China*, World Scientific 2004.

[2] G. Cœdès. *A propos de l'origine des chiffres arabes*, Bull. School of Oriental Studies **6**(2), 1931

[3] G. Cœdès. *The Indianized States of Southeast Asia*, Univ. of Hawaii Press, 1968.

for 10^k, $k = 0, 1, \ldots, 6$. The Aztecs had symbols for 1, 10, 20, 100, 200, 300, 400, and 8000. Some Greek and Hebrew systems used letters of the alphabet like in `\alph` to denote numbers but combine it with a decimal system to enlarge the representable numbers. For example the Hebrew latter have 22 letters to denote 1-9, 10-90, and 100-400. They added 5 more letters for 500-900.

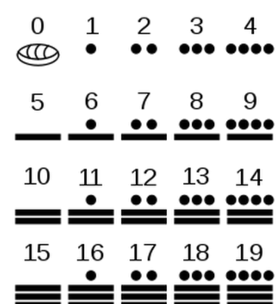


Babylonian numbers

Of course the oldest and really simple systems had only one symbol: a bar, and one had to count tally marks. If we group them in packages of five or ten, these packages can be represented by a second symbol. The Babylonian cuneiform system represented a unit and a ten, some Chinese and Mayan systems also had two marks: a unit and a group of five. These were enough to form numbers from 1 to 9 or 10, but the ingenuity came from the way they were composed to form larger numbers without inventing new symbols. Sometimes decorations were added to the symbols like dots or bars to multiply them with a power of 10. Archimedes had a way to describe really large numbers 'exceeding the the number of grains of sand in the whole universe'. First order were myriad (10^4) and a myriad-myriad (10^8). Putting order upon order and periods upon periods and unit upon units, he had for example a myriad-myriad units of the myriad-myriadth order of the myriad-myriadth period was actually $10^{8\times10^{15}}$. He wanted to be able to count all the grains of sand on the beach or even in the world. Knuth's up-arrow notation for iterated exponentation, now common in Ramsey theory, uses a similar idea, but is more structured.

Since we are used to our system, it is obvious that the positional decimal system has won, but this did not come about overnight. The problem was that the zero was not part of the counting system. If you count horses, you will count at least one horse, but you cannot count horses when there is no horse around. So one left a space at a certain position if needed, but it was difficult to distinguish between one, two or three spaces. Thus some placeholder was invented to mark the spaces like the Babylonians or the Maya did. Note that as a placeholder it did not appear at the end of a number. It was definitely not considered to be a number until Bramagupta ($\pm$ 628 AD) had it with the arithmetic rules and negative numbers. Stone tables of Gwalior (South of Dehli, 876 AD) had a round indication of zero and copper plates go back to the 6th century but these could be forgeries. But Kaplan discusses many other possible notations of what can be indicated as zero or perhaps as a place holder. There was a dot, that could also be a separator. Some zeros look like a theta. Or other possibilities that look less familiar to us. According to Mazur, the positional system came to Europe via the Arab translation in Al-Khwārizmī's *al-jabr* (825 AD) together with the Hindu-Arab numerals. Fibonacci's *Liber abbaci* helped distributing the computational rules into Europe did miss out the number zero and spoke of only 9 digits. The zero as a number was re-introduced some 200 years later in work by N. Chuquet while solving quadratic equations. Although the Mayan Haab

Haab calendar



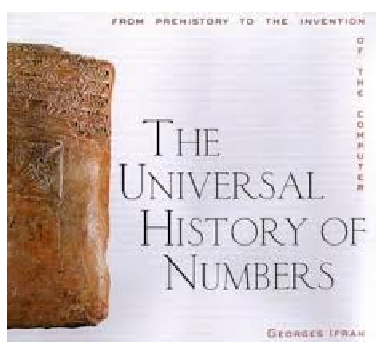
Aztec numerals

calendar had months of 20 days, numbered from 0 to 19. So they certainly had the zero. It had been uttered that the Hindu got the numbers and/ the zero from the Babylonians through the Greek astronomy. They had certainly ways to talk about zero: *kha* (space), *ambara* (sky) *sunya* (empty). There are also many speculations about the origin of the symbol (something circle-like) to denote a zero. Some say that it is the mark left in the sand when stones were used for counting and a stone was taken away.

European	0	1	2	3	4	5	6	7	8	9
Arabic-Indic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Eastern Arabic-Indic (Persian and Urdu)	۰	۱	۲	۳	۴	۵	۶	۷	۸	۹
Devanagari (Hindi)	०	१	२	३	४	५	६	७	८	९
Tamil		௧	௨	௩	௪	௫	௬	௭	௮	௯

Kaplan goes on discussing the early development of mathematics, once the Hindu system for numbers were introduced with the notation for the unknown, the way equations were solved, and how Leibniz and Newton developed calculus. For those who are not familiar with mathematical history, it will probably be astonishing to learn how recent our current notation of mathematical formulas is. It is almost unbelievable that mathematics had evolved till the 15th-16th century on a rhetorical basis, not even a plus sign or an equal sign existed so that equations were described with plain words. Since around 1500 symbols were introduced increasingly and we see an exponential increase in mathematical knowledge ever since. So how did one arrive at these symbols? Where did they come from? What were the mechanisms that shaped them into their now familiar form? Mazur gives an entertaining history of this evolution. In a first part he deals with the notational systems of the numerals as was explained above. Then he considers the usual mathematical symbols like plus, minus, exponentials, square root, etc. in an algebraic context. In a third part he leaves history behind and ponders on the influence of our symbolic notation on the psychology of mathematicians, how symbolic patterns are stored in our brain and how formulas trigger unconscious associations. In retrospect, this may explain how symbolic notation has influenced the evolution of mathematics.

Georges Cœdès

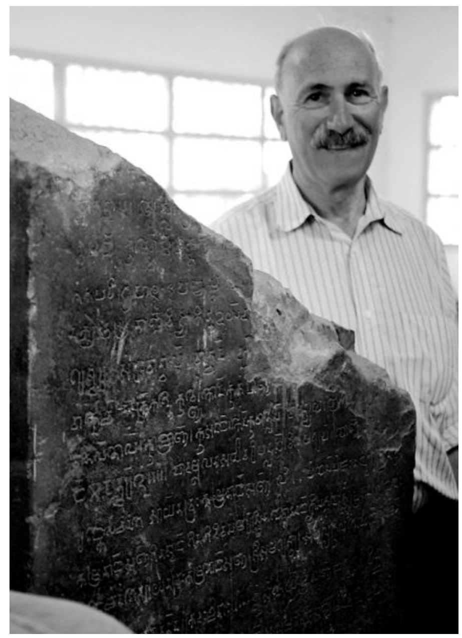


George Ifrah's book

George Ifrah

But let me spend some more words on the book by Aczel. It is all about mathematics and re-establishing the work of Cœd‘es mentioned above and in particular about finding out the whereabouts of the stone stele in Cambodia that was numbered K-127 (a prime number) by Cœdès on which he found the proof that the zero was known to the Indians before the Greek used to denote it. Aczel's father was a captain on a cruise ship and the children were often looked after by Laci (pronounced lotzi), the personal stewart of the captain. This Laci raised Aczel's interest in numbers and mathematics. So he tells the usual stories about the number systems that can also be found in the other books. Later, when he had become a mathematician, he wanted to find out more about the origin of the Hindu-Arabic number system and travels to India, trying to find out why zero (and infinity) more naturally could arise in a cultural climate of the East. Hinduism, Jainism, Buddhism (Buddha was a mathematician), all differ from the way one used to think in the West with out yes or no, true or not true. They have 4 states: true, not true, both true and untrue, and neither true nor untrue (tetralemma). So the 3rd and the 4th may be the seeds that gave rise to the infinite and the nothing (the zero). Aczel even tries to connect the explicit sexual imaging in some of the Hindu temples with the mathematics. In the West the adagio was that nothing could come from nothing. We had to wait till Cantor defined set theory and hence the number system starting with the empty set for zero, then the set containing only the empty set, then the set with the two previous sets as elements, etc. He finally arrives at infinity and the different orders of infinity.

K-127, the inset = 605
The inscription says: "The Chaka era reached year 605 on the fifth day of the waning moon…" We know that in Cambodia the Chaka era began in the year 78 AD. Thus the date of this zero is 605 + 78 = 683.



Aczel at the moment he found K-127

He then learned about the book by Georges Ifrah *The Universal History of Numbers* who cites the work of Cœdès (1886-1969), something also Laci had already pointed out to him, but that he had forgotten. The K-127 that Cœdès had described that contained the oldest known notation for zero as a dot had been in the Cambodian national museum. However when the Red Khmer took over many artifacts were destroyed and nobody knew what had become of the K-127. So Aczel took it as his mission to rediscover the stele. So much of his book is about his adventurous traveling to the Southeast Asian countries in search of K-127. I think I will not take away the thrill of the story when I disclose that he finally was able to find it eventually in 2013. However right at that moment of euphoria, two Italian ladies enter the emporium to pick up at random some piece for their restoration students to practice on. Aczel over-enthusiast just now that he reaches the end of his quest, tells them about the stele. Unfortunately, as a result they insist on having K-127 for their students to test their abilities, much to the shocking dismay of Aczel who considers this as the most fundamental finding for mathematics that had to be conserved and handled with utmost care. The rest of the book consists of his efforts to save K-127 from the supposedly malicious intentions of the Italians. How that ends, I will not disclose.

A. Bultheel

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