

# Newsletter

BELGIAN MATHEMATICAL  
SOCIETY

# 122, March 15, 2019

Comité National de Mathématique CNM

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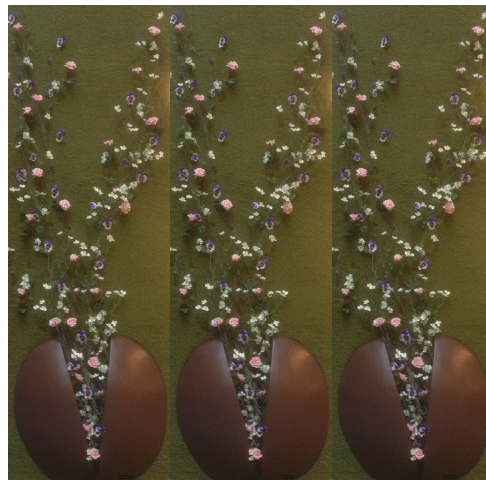
NCW Nationaal Comité voor Wiskunde

**Newsletter of the Belgian Mathematical Society  
and the National Committee for Mathematics**

ULB Campus Plaine, C.P. 218/01,  
Bld du Triomphe, B-1050 Brussels, Belgium

Website: [bms.ulb.ac.be](http://bms.ulb.ac.be)

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The next edition of this newsletter will appear on May 15th, so from now till May 10th all content is welcomed very much at [wendy.goemans@kuleuven.be](mailto:wendy.goemans@kuleuven.be).

## The President's Foreword

Dear members of the BMS,

Our society was founded on  $\pi$ -day 1921 by Théophile De Donder. More than 30 times  $\pi$  years later, the society is still alive and thriving and we need your help to keep it relevant for the next  $30\pi$  years. So please, follow our publications, join us on Facebook, encourage your students, colleagues, friends and parents to become members, and most importantly communicate with us to tell us what the BMS can do for you.

Yvik Swan,  
BMS President

## L'Oreal-UNESCO international award for women in science

Ingrid Daubechies and Claire Voisin received the L'Oreal-UNESCO international award for women in science. See the website

<https://en.unesco.org/news/21st-edition-loreal-unesco-international-awards-women-science>

Congratulations!!

## 1 News from the BMS & NCM

### 1.1 Save the date: Godeaux Lecture on June 3, 2019 KU Leuven Campus Kulak Kortrijk.

The next Godeaux Lecture will be given by prof. John Guaschi (université de Caen Normandie, France) during the first day of the conference "Nielsen Theory and Related Topics".

In fact the programme of the first day of the conference will consist of 5 talks accessible for a large audience. It will be possible to register for the first day only. This information will be on the website of the conference soon,

<https://www.kuleuven-kulak.be/nielsen>

### 1.2 Bulletin of the Belgian Mathematical Society - Simon Stevin

In December 2018 Volume 25, Number 5 of the Bulletin of the Belgian Mathematical Society - Simon Stevin appeared with the following table of contents:

- **Adara M. Blaga** Almost  $\eta$ -Ricci solitons in  $(LCS)_n$ -manifolds. 641–653.
- **Stefano Isola, Riccardo Piergallini** On the generic triangle group and the free metabelian group of rank 2. 653–676.
- **Mohammad Ansari** Strong topological transitivity of some classes of operators. 677–685.

- **Hossein Javanshiri, Mehdi Nemati** Invariant  $\varphi$ -means for abstract Segal algebras related to locally compact groups. 687–698.
- **Abdullah Atmaca, A. Yavuz Oruç** Counting Unlabeled Bipartite Graphs Using Polya’s Theorem. 699–715.
- **Serge Nicaise** The Helmholtz decomposition in weighted  $L^p$  spaces in cones. 717–728.
- **Hadis Pazandeh, Fereshteh Sady** Function Spaces and Nonsymmetric Norm Preserving Maps. 729–740.
- **Bruce R. Corrigan-Salter** Higher Order Hochschild (Co)homology of Noncommutative Algebras. 741–754.
- **Amol Sasane** A potpourri of algebraic properties of the ring of periodic distributions. 755–776.
- **Khalil Ayadi, Awatef Azaza, Iheb Elouaer** On the multiplication by a polynomial of bounded continued fraction over a finite field. 777–792.

For table of contents of previous issues, see <https://projecteuclid.org/all/euclid.bbms>.

Remember, as a member of the BMS you can ask for electronic access to all electronically available issues of the bulletin, if you don’t have a login yet, contact [pcara@vub.ac.be](mailto:pcara@vub.ac.be).

## 2 Meetings, Conferences, Lectures, ...

### 2.1 Meetings and conferences

#### 2.1.1 March 2019

**Ecole doctorale**

**Services d’Analyse Mathématique et de Probabilités et Statistique**

**March 21, 2019**

**Mons**

See the announcement at the end of this newsletter.

#### 2.1.2 April-May 2019

**Kulak - VUB Algebra days**

**April 25 (Kortrijk) and May 10 (Brussels), 2019**

We are organizing two meetings with talks in Kortrijk by the ‘Algebra and Incidence Geometry’ research group of VUB and in Brussels by the ‘Algebraic topology and group theory’ research group of KU Leuven Kulak. The goal of these meetings is to start new collaborations, so there will be plenty of time for social interactions as well. More information, including speakers, titles and abstracts, will be available soon on the web page

<https://www.kuleuven-kulak.be/nl/onderzoek/key-areas/onderzoeksgroepen/zuivere-wiskunde-en-wiskunde-onderwijs/seminar/kulakbrussels/>.

### 2.1.3 May 2019

## 30<sup>th</sup> Congress MATH.en.JEANS

May 3-5, 2019

Louvain-la-Neuve

MATH.en.JEANS is an organization, founded in France in 1989, whose aim is to develop contacts between researchers in mathematics and secondary school students by offering those the opportunity to develop an authentic research activity. Concretely, the researchers propose various topics of investigation, on which the pupils work by small groups since the beginning of the school year. The work is then developed during the year, punctuated by tight regular contacts with the researchers. The ultimate goal is to prepare a presentation for the annual congress. In 2019, twelve such congresses will be held in Belgium, France, Romania, la Réunion and the USA. The Belgian congress will be in French. It will take place at UCLouvain in Louvain-la-Neuve from Friday May 3 till Sunday May 5. About 400 participants are expected from Belgium, Luxemburg and the French region 'Grand-Est'. Each group of students will give a 15 minutes presentation of their work, and hold a stand at the forum (poster session). In addition, three lectures targeted at a general audience will be given by the **plenary speakers**:

- Julie DE SAEDELEER
- Mickaël LAUNAY (alias Micmaths, see <http://www.micmaths.com/>)
- Nathan UYTENDAELE (creator of *La Statistique expliquée à mon chat*, see [https://www.youtube.com/channel/UCWty1tzwZW\\_ZNSp5GVGteaA](https://www.youtube.com/channel/UCWty1tzwZW_ZNSp5GVGteaA))

The location of the Congress is Place des Sciences, 1 in Louvain-la-Neuve. The schedule is as follows:

- Friday, May 3
  - 13h00 – 14h00: Registration (Hall des Sciences)
  - 14h00 – 16h20: Presentations by the students
  - 16h45 – 17h45: Plenary talk by Mickaël Launay
  - 18h00 – 20h00: Dinner
- Saturday, May 4
  - 9h15 – 12h30: Forum and presentations by the students
  - 12h30 – 14h30: Lunch
  - 14h30 – 15h: Plenary talk by Nathan Uyttendaele
  - 15h00 – 16h15: Forum
  - 19h45 - 21h00: Improvisation show
- Sunday, May 5
  - 9h15 – 11h30: Presentations by the students
  - 11h45 – 12h30: Plenary talk by Julie De Saedeleer

Conference website: <https://www.mathenjeans.be/congres2019>

Contact person: Pierre-Emmanuel Caprace ([pierre-emmanuel.caprace@uclouvain.be](mailto:pierre-emmanuel.caprace@uclouvain.be))

Sponsors: UCLouvain, ULiège, Belgian Mathematical Society, Service Public Wallonie

#### 2.1.4 July 2019

### International Conference on Differential & Difference Equations and Applications 2019

July 1-5, 2019

Lisbon (Portugal)

See all information at <https://sites.google.com/site/sandrapinelas/icddea-2019>

#### 2.1.5 September 2019

### Summer School in Algebra and Topology

September 11-14, 2019

Louvain-la-Neuve (Belgium)

It is a pleasure to announce the second edition of the Summer School in Algebra and Topology, which will take place in Louvain-la-Neuve from Wednesday the 11th until Saturday the 14th of September 2019.

The summer school will consist in four intensive courses addressed to Master and PhD students, which will also be of interest to more experienced researchers in mathematics. This event is part of a joint project of UCLouvain with the universities of Coimbra, Padova and Poitiers, promoting research collaboration within the Coimbra Group.

The invited speakers of this summer school will be professors Silvana Bazzoni (Padova), Marino Gran (Louvain-la-Neuve), Sandra Mantovani (Milano), and Jorge Picado (Coimbra). The titles of their courses are:

- Contramodules and their applications - Silvana Bazzoni
- Categories for universal algebra - Marino Gran
- Categorical commutator theory - Sandra Mantovani
- Frames and locales: topology without points - Jorge Picado

To register, please send a message to Mme Carine Baras [carine.baras@uclouvain.be](mailto:carine.baras@uclouvain.be) mentioning your academic status (Master or PhD student, post-doc, professor, etc.) and affiliation before the 15th of June 2019. If you wish, she can also help you with booking one of the rooms we reserved at the student residence "Le Relais" and at the hotel "Ibis Style"; in that case, please contact her as soon as possible.

There is limited funding to partially cover local expenses for students. If you wish to apply for this, then please send me an up-to-date CV and a reference letter before the end of March 2019.

Further information is available on the poster at the end of this newsletter and on the website

<https://uclouvain.be/en/research-institutes/irmp/summer-school-in-algebra-and-topology-2019.html>

With best regards,  
Tim Van der Linden ([tim.vanderlinden@uclouvain.be](mailto:tim.vanderlinden@uclouvain.be))

## 2.2 Seminars

### 2.2.1 Algebra section of KU Leuven

For the upcoming seminars, see the calendar at

<https://wis.kuleuven.be/algebra/calendar-sem-ntag>

## 3 PhD theses

### Extensions of the Pascal triangle to words, and related counting problems

**Manon Stipulanti, Doctorante et boursière FRIA (FNRS)**

**Département de Mathématique (B37)**

**Université de Liège**

*April 2, 2019 at 16:30*

*Auditorium 02 at the Department of Mathematics (B37)*

*Université de Liège, Allée de la Découverte 12, 4000 Liège*

Thesis advisors: Émilie Charlier (Université de Liège) and Michel Rigo (Université de Liège)

#### Summary

The starting point of this thesis is the relation between the Pascal triangle, which is made of binomial coefficients of integers, and the corresponding Sierpiński gasket. The former is named after the French mathematician Blaise Pascal who lived during the 17th century, though it appeared centuries before in different parts of the world. On its side, the Sierpiński gasket takes its name from the Polish mathematician Waclaw Sierpiński who lived during the 20th century. Those two mathematical objects have been extensively studied through the ages and worldwide. They exhibit connections with many different scientific areas such as fractal theory, dynamical systems, automata theory, logic, arithmetic, enumerative combinatorics,  $p$ -adic topology and  $p$ -adic analysis.

Several generalizations and variations of the Pascal triangle do already exist. For instance, they are studied with arithmetical and combinatorial viewpoints, dynamical ones or analytical ones. In this dissertation, we define new extensions by means of binomial coefficients of words, which expand the classical notion of binomial coefficients of integers as explained below. Let  $A$  be a finite alphabet, *i.e.*, a finite set of characters or letters. A word over  $A$  is simply a sequence of letters belonging to  $A$ , which can be either finite or infinite. The binomial coefficient  $\binom{u}{v}$  of two finite words  $u$  and  $v$  over  $A$  is the number of subsequences of  $u$  that exactly match  $v$ . Observe that if  $a$  is a letter, then the binomial coefficient of  $a^n$  and  $a^k$ , which respectively represent  $n$  and  $k$  letters  $a$  glued together, is the number of ways to select  $k$  letters  $a$  among  $n$  available letters  $a$ , which is exactly  $\binom{n}{k}$ .

This thesis is centered at the Pascal triangle, binomial coefficients, numeration systems and related questions, and is articulated around three parts.

The first part is dedicated to extensions of the Pascal triangle and the Sierpiński gasket to various sets of words (languages) associated with different numeration systems. The so-called Parry numeration

systems, based on Parry numbers, form a well-known and widely studied class of numeration systems containing the integer base numeration systems and the Zeckendorf numeration system based on Fibonacci numbers. Within this rather general setting, we study the corresponding Pascal triangle and Sierpiński gasket, and we transport the existing link between them.

The second part is concerned with particular sequences extracted from generalized Pascal triangles. They count non-zeroes binomial coefficients on each row of a given Pascal-like triangle. We study their regularity and their automaticity with respect to different numeration systems. As it is a challenging problem to determine what are the “best” data structures for reasoning with subsequences, we present a new method based on trees to prove the regularity property. It allows us to tackle the exotic case of Fibonacci-regular sequences. It is worth noticing that regular sequences in the Fibonacci framework are not so easy to find in the literature, which endows a certain bonus to this work.

In the third and last part, we establish the asymptotics of the summatory functions of the sequences considered previously. Traditional methods to deal with summatory functions of regular sequences are on an algebraic or analytic side. Our contribution is to develop a new systematic method to obtain such asymptotic estimates. It is based on the construction of a convenient and exotic numeration system, leading to non-standard representations. In the aftermath of this original technique, we treat the case of the Zeckendorf numeration system and the Fibonacci numbers, which was never done before.

## 4 News from the universities

### 4.1 VUB: Doctor Honoris Causa 2019

**Figures that matter**

**April 2, 2019, 11am**

**VUB, Etterbeek campus, Brussels**

Figures that matter were the terrain of the Swedish statistician **Hans Rosling** who died in 2017 and whose son, Ola Rosling, and daughter-in-law, Anna Rosling Rönnlund, have completed his worldwide bestseller *Factfulness* (Fact Knowledge, 2018).

**Karine Chemla** (France) is a historian of mathematics, **Freddy Van Oystaeyen** is a Belgian pioneering mathematician, **Padmanabhan Seshaiyer** (US) is also a mathematician, but above all a champion and pioneer of better STEM education (Science, Technology, Engineering & Mathematics).

**Brian Durie** is a world authority in Multiple Myeloma (bone marrow cancer) and is, **together with his wife Susie Lavitt Durie**, founder of the International Myeloma Foundation.

Finally, there are honorary doctorates for the Dutch physicist and science communicator **Robbert Dijkgraaf** and political cartoonist **Gerard Alsteens** (GAL).

For all information, see

<https://www.vub.ac.be/dhc2019/>

## 4.2 Platform Wiskunde Vlaanderen

Since a few years, a small group of volunteers have been working on the establishment of a "Platform Wiskunde Vlaanderen" that should unite anyone in Flanders that get into contact with mathematics. Without polarisation between "pure and applied", "science and technology", "useful or beautiful" or anything else. In that time, we made a couple of texts that should express a unifying vision of what mathematics is and means for society. We also tried to collect support for the idea with policy makers. That these efforts take place at the Flemish level, instead of the Belgian one, is a consequence of the fact that education and research reside at the level.

We thought it would be a good idea to use " $\pi$ -day" to get some public attention for mathematics and for this initiative. To this end, we formulated a mission statement (in Dutch), which can be found at <http://www.platformwiskunde.be/verklaring>. That mission statement was signed by the Flemish rectors, the chair of the education organisations (onderwijskoepels), the association of mathematics teachers VVWL, the Belgian Mathematical Society (BMS) and a number of prominent non-mathematicians (company leaders and media figures). We would like to ask your support for this message.

Hopefully, there will be some media attention for that mission statement. It would be good if the mathematical community would support this unifying initiative. We would therefore like to ask if you could forward this message to relevant people in your network with the question if they would like to support this message. Anyone that wants to sign can do so by sending a mail to [verklaring@platformwiskunde.be](mailto:verklaring@platformwiskunde.be) with name and affiliation, which will then be added to the list.

With this message, we would like to inform you about the events that will take place on  $\pi$ -day:

- launch of the website <https://platformwiskunde.be> with a number of texts
- a double interview in the news paper "De Standaard" with Ann Dooms and Giovanni Samaey
- a short item on VRT tv, of which the format is still unclear.
- Universiteit van Vlaanderen provides a dedicated page with all mathematical lectures.
- who knows what else will happen ;-)

Of course, all these events don't mean we have a working "Platform". On the contrary: it all still has to start. Still, we are already going public to try to gain some momentum. The goal now is to collect wide support for the idea, such that we can also reach a broader pool of people to work on specific initiatives. If you are interested in working out initiatives for education, research, innovation or mathematics communication, we would love to hear from you. Please send a message to [contact@platformwiskunde.be](mailto:contact@platformwiskunde.be).

Kind regards,

Bart De Moor (KU Leuven), Ann Dooms (VUB), David Eelbode (UAntwerpen), Paul Igodt (KU Leuven en VWO), Eric Jaspers (VUB), Sorin Pop (UHasselt), Giovanni Samaey (KU Leuven), Bert Seghers (Jonge Academie), Stefaan Vaes (KU Leuven), Joris Van der Jeugt (UGent)

## 5 History, maths and art, fiction, jokes, quotations ...

Following are  $\pi$ -day contributions by Paul Levrie and two reviews of Adhemar Bultheel on the geometrical beauty of plants and on a thriller about a mathematical character.



# pi-trivia

Did you know that ...

*pi*

- ... today is  $\pi$ -day?  
Why? Because in America they write 3/14 for the date of today, March 14, and 3.14 is an approximation to the number  $\pi$ .  
Today you should eat pie, as everyone does. Or you should buy some pie for someone!
- ... the number  $\pi$  is a mathematical constant that is the ratio between the circumference of a circle and its diameter? The first 500 decimal digits of  $\pi$  are given by:

3.141592653589793238462643383279502  
88419716939937510582097494459230781  
64062862089986280348253421170679821  
48086513282306647093844609550582231  
72535940812848111745028410270193852  
11055596446229489549303819644288109  
75665933446128475648233786783165271  
20190914564856692346034861045432664  
82133936072602491412737245870066063  
15588174881520920962829254091715364  
36789259036001133053054882046652138  
41469519415116094330572703657595919  
53092186117381932611793105118548074  
46237996274956735188575272489122793  
818301194913

Note that the year 2019 appears prominently between the first digits of  $\pi$ .

- ... the  $\pi$ -record of November 2016 is still standing? In that year Peter Trüb calculated  $\pi^e$  trillion digits of  $\pi$ . This means 22,459,157,718,361 digits. It took the computer months to finish the job: The calculation started on July 29, and ended November 11. Note that the record for a single hexadecimal digit has been broken in the same year: D. Takahashi calculated the  $10^{17}$ -th hexadecimal digit of  $\pi$ . It took his computer 641 hours and 29 minutes (verification included).
- ... there really is a relationship between the number  $\pi$  and the Platform Wiskunde Vlaanderen that's going public today (see elsewhere in this Newsletter)? It sounds like a cry for help:  
 $\text{PIIIIII} \implies \text{PWV}$
- ... there are many infinite expressions giving an exact value in some way containing

$\pi$ ? My favourite one still is John Wallis's product formula for  $\pi$ , which dates from 1655.

This is it:

$$\frac{\pi}{4} = \frac{2 \cdot 4}{3 \cdot 3} \cdot \frac{4 \cdot 6}{5 \cdot 5} \cdot \frac{6 \cdot 8}{7 \cdot 7} \cdot \frac{8 \cdot 10}{9 \cdot 9} \cdots$$



- ... many of these infinite expressions are series: Sums with an infinite number of terms? The most famous one is probably the series with the many names (Gregory - Leibniz - Madhava - ...):

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

There are not that many series known for which the sum contains  $\pi^5$ . Here are two of them:

$$1 + \frac{1}{2^{15}} \sum_{n=1}^{\infty} P(n) \frac{(n-1)!^{10}}{\left(\frac{1}{2}\right)_n^5 \left(\frac{1}{2}\right)_{n+1}^5} = \frac{\pi^5}{2^5}$$

and

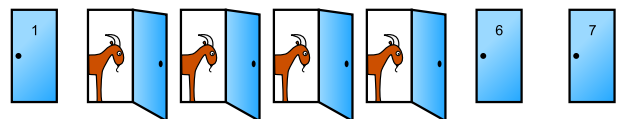
$$1 - \frac{1}{2^5} \sum_{n=1}^{\infty} P(n) \frac{\left(\frac{1}{2}\right)_{n-1}^5 \left(\frac{1}{2}\right)_n^5}{n!^{10}} = \frac{2^5}{\pi^5}$$

with  $P(n)$  in both series equal to  $1280n^8 - 640n^6 + 160n^4 - 20n^2 + 1$ . Here we use the notation:

$$(a)_n = a \cdot (a+1) \cdots (a+n-1),$$

the so-called Pochhammer-symbol.

- ... in the Monty Hall problem with  $n$  doors ( $n$  odd) (if you don't know this problem: Google "Monty Hall") it's always better to change doors, especially in the case that the quizmaster keeps opening doors revealing a goat:



The probability  $p$  of winning in that case is given by an expression containing the number  $\pi$ :

$$p \sim \sqrt{\frac{\pi}{2n}}.$$

This is an immediate consequence of Wallis product formula for  $\pi$ , and one of the many

instances where Wallis's formula plays a crucial role. (Thank you, Stijn.)

- ... are many good and interesting approximations for  $\pi$ ? Here are some:

$$\pi \approx \frac{6}{5}\phi^2$$

$\phi$  is the golden ratio.

There's also this one:

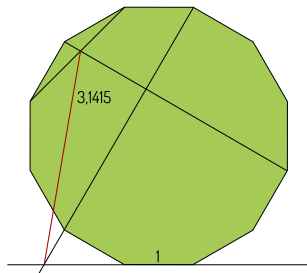
$$\sqrt[4]{\pi} \approx \frac{3}{14} \sqrt{\frac{193}{5}}$$

Just try them out!

This is also nice:

$$(3, 1, 4) \equiv (1, 5, 9) + (2, 6, 5) \pmod{10}$$

And recently it was shown that you can get a good approximation (that is constructible) for  $\pi$  using the diagonals of a regular dodecagon:



- ... the following lyrics should be sung using the tune 'My Heart Will Go On' from the movie Titanic?

Pi Will Go On

Ev'ry time I circle, I see you, I feel you  
That is how I know you go on:  
Trillions of digits, no pattern to show us,  
You have come to show you go on.

CHORUS: Find  $\pi$  however we try,  
We will see that  $\pi$  does go on.  
Before, one guy called it 4,  
But I know in my heart that  $\pi$  does go on and on.

22/7 might seem more pleasant,  
But  $\pi$  never ends or repeats.  
 $\pi$  is a ratio, but never a fraction:  
This number really is neat!

CHORUS: Find  $\pi$  however we try,  
We will see that  $\pi$  does go on.  
Arctan however you can,  
But Lambert showed that  $\pi$  will go on and on.

- ... some people have more luck with their new license plates (than I had)?



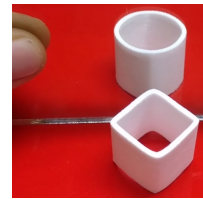
(Bedankt, Katrien!)

- ... the following puzzle is related to the number  $\pi$ ? Each letter corresponds to a unique digit. Try to find them to make the sum work. What has all this to do with  $\pi$ ?

$$\text{SIX} + \text{SIX} + \text{SIX} = \text{NINE} + \text{NINE}$$

(From Martin Gardner's The Magic Numbers of Dr. Matrix (1985)).

- ... a mirror makes 'squaring the circle' possible?



- ... alongside the highway in Ghent there's a building with quite some decimal digits of  $\pi$  on its facade? Don't try to look for them if you're passing by with your car: This is much too dangerous.
- ... with these ingredients celebrating  $\pi$ -day can't be a problem?



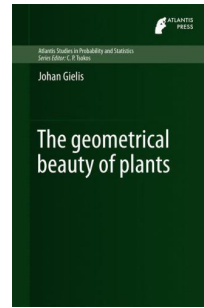
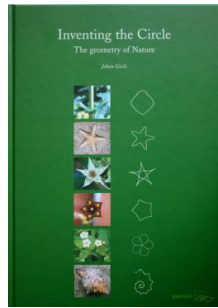
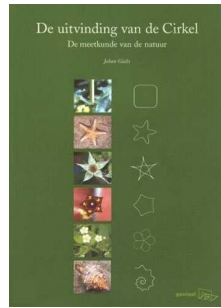
(Thanks Erwin and Karen!)

- But remember: Pi-day is just a fake holiday invented by math companies to sell more math!
- And then there's this:



**Inventing the circle: the geometry of nature** Geniaal, 2003, isbn 9062167920 | **De uitvinding van de cirkel: de meetkunde van de natuur** Geniaal 2001, isbn 9789062157921, 188 p.  
**The geometrical beauty of plants**, Atlantis Press, 2017, isbn 9789462391505, xxv+229 p.

Johan Gielis is a botanical engineer from the University of Antwerp. He started using supercircles and superellipses in the 1990's while studying bamboo whose cross section had this kind of shape.

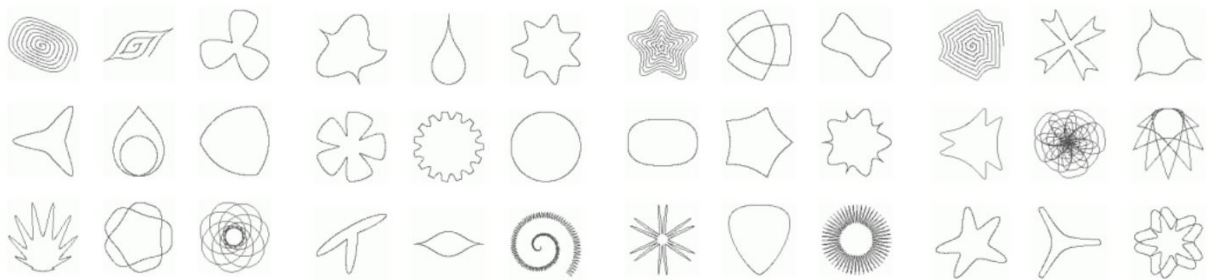


Johan Gielis

These curves are described by  $x^n + y^n = R^n$ , or in polar coordinates  $\rho(\theta) = [\cos(\theta)^n + \sin(\theta)^n]^{-1/n} R$ . For  $n = 2$ , this is just  $\rho(\theta) = R$ , the usual Euclidean circle with radius  $R$  but it can of course easily be generalised to ellipses. These shapes were known and used by the Danish mathematician Piet Hein. Later, he generalised his formula to describe many other shapes of stems, leaves, flowers, etc. In several steps he arrived eventually at a formula that he called a superformula of the form

$$\rho(\theta) = \left[ \frac{1}{a} \left| \cos \left( \frac{m_1 \theta}{4} \right) \right|^{n_2} + \frac{1}{b} \left| \sin \left( \frac{m_2 \theta}{4} \right) \right|^{n_3} \right]^{-1/n_1} R(\theta). \tag{1}$$

The parameters  $a$  and  $b$  give the “elliptic touch”,  $m_1, m_2$  somehow defines how many “peaks” or corners the result shall have, and the  $n_i$  parameters govern the size and the convexity of the result. The parametric function  $R(\theta)$  was originally a constant, but if for example  $R(\theta) = \theta$ , this results in spirals. Some examples:



These were taken from [www.procato.com/superformula/](http://www.procato.com/superformula/) where you can experiment with the parameters interactively. For more interactive sites and octave code see the superformula on wikipedia. Of course it is not so difficult to generalise this to 3D which gives the most bizarre and diverse shapes: see this link for a YouTube video. Gielis took a patent and founded Genicap, a software company to exploit the possibilities of the formula as a subsidiary of his company *Geniaal bvba*. His first book on the topic was published in Dutch (2001) and later in English (2003) by Geniaal.

That book, *Inventing the circle*, is richly illustrated and introduces step by step the superformula starting from the circle. Many illustrations are from biology and especially from plants. In particular the bamboo example gets much attention, but also many shapes of leaves and flowers from other plants. Also eggs, seeds, cells, algae, pollen, fruits, sea shells, starfish, phyllotaxis and morphogenesis, you name it.

His conclusion is that everything is just a circle if you are willing to use a flexible anisotropic measure of distance. This is indeed what this formula is doing. If we use an isotropic fixed distance measure, then all points at a rigid distance  $R$  give the usual Euclidean circle with radius  $R$ . If we let the measure vary with the angle  $\theta$ , then we can obtain all the shapes generated by the superformula.

In the first book we learn to know Gielis as the bio-engineer who discovers this mathematical formula and becomes euphoric about its flexible applicability and its beauty in all its simplicity. In his second book, he has obviously grown into the subject, and we meet more of a mathematical engineer. His formula, which is now called the Gielis transform, is still applied to plants, but he has now discovered the connection with

many other applications and branches of mathematics, so that some chapters are more mathematical than biological. Although the title of the first book is more mathematical and the second title refers explicitly to plants, for the contents it is rather the other way around. His recent book is some kind of summary of all the mathematical and applied papers that he (co-)authored in the 15 years after his first book.



Not only is there more mathematics, there is also more historical background and even philosophy. He has subdivided the book into 6 parts. According to the commentaries of Proclus on Euclid’s elements, this is how a theorem is proved in the Platonic tradition. One has to start with the *Propositio*, which is the announcement and exploration of an idea. It is followed by the *Expositio*, the particular announcement of the claim. To give the demonstration one has to collect the predicates in the *Determinatio*, then apply the postulates and prepare in the *Constructio* for the actual proof, which is given in the *Demonstratio*. The *Conclusio* is the summary of what has been proved.

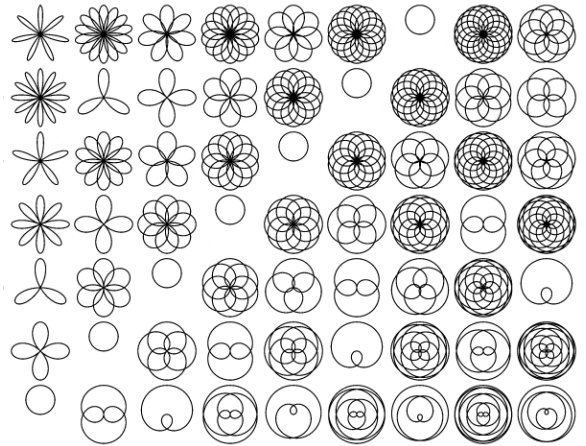
In his *Propositio* Gielis reflects on the difference between a pure mathematical object (which never exists in nature) and what is observed and how, in the mind’s eye, mathematical models are used to describe reality, neglecting the “imperfections”. Mathematical elements are explored like arithmetic and geometric mean, and the idea of a flexible elastic measure that depends on the direction in which one is measuring. The *Expositio* introduces the ideas on how to generalise the classical definitions of arithmetic and geometric mean by giving weights to the components that are averaged like  $AM_{i/n} = ((n - i)a + ib)/n$  and  $GM_{i/n} = \sqrt[n]{a^{n-i}b^i}$  and the harmonic mean  $HM_{i/n} = nab/(ia + (n - i)b)$ . He links this to Pascal’s triangle, Newton’s fluxions, the monomiality principle (all polynomials are transformed monomials using derivative ( $P$ ) and multiplication ( $M$ ) operators) and the Heisenberg-Weyl algebra. Here Gielis seems to be dragged away a bit in his enthusiasm because at some points he states the commutation relation  $[P, M] = 1$  without explaining what the notation means, one has to look up Dattoli’s papers for that, and on page 37 he writes “Sine and cosine are examples of simple polynomials,...”. Humph? The definitions of the means lead to Lamé curves (described by generalisations  $x^n + y^n = R^n$  of the Pythagoras theorem) and the 3D analogs. And finally the superformula which is ‘transformed’ (pun intended) into the Gielis transform. On page 5 it is announced the Gielis is an acronym, but one has to wait till page 118 where it is explained as ‘Generalised Intrinsic and Extrinsic Lengths In Submanifolds’).



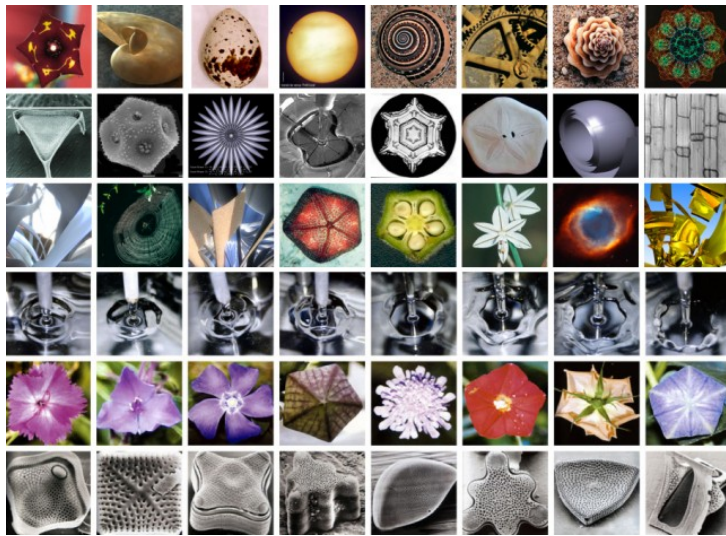
genicap logos designed by Boy Bastiaens & Albert Kiefer

In the *Determinatio* even more general versions are considered when the  $\theta$  in the arguments of sine and cosine in the formula (1) are replaced by functions  $f_i(\theta)$  and  $f_2(\theta)$ . Also the exponents could be replaced by functions, or a set of parameters is chosen for  $i = 1, 2, \dots, N$  and then these formulas are used in a linear combination of the corresponding  $\rho_i(\theta)$ . The latter is then some general form of a polynomial or even a Fourier expansion of a curve. Of course again 3D generalizations are feasible. One may generate  $m$  dimensional manifolds in an  $n$  dimensional space and Generalised Möbius-Listing (GML) bodies combining a degree of symmetry and a number of twists. The possibilities are quasi infinite. The advantage is that in this way the most complex objects are completely characterised by

a simple set of values for the parameters. Gielis calls these oligomials (oligo = few). This can be used in a definition for geometric or topological complexity. All Gielis curves are equally simple. Self-intersecting curves and surfaces are also possible which can represent for example complex dynamical systems. The mathematical aspects involve Lindenmayer L-systems and fractals. R-functions (named after Rvachev) are real functions whose sign can only change if the sign of one of its arguments changes. The sign can be interpreted as true or false and thus there is a connection with logic and set theory. The flexible measure of distance is linked to the Lorentz transform and the special relativity theory in a space-time continuum. The AM and GM means return in a discussion of Finsler-Riemann-Lamé-Minkowski (FRLM) geometry and the cosmological metrics à la Friedman-Lemaître-Robertson-Walker (FLRW). The Rhodonea curves of Guido Grandi (1722) are spirograph-like curves described by  $\rho(\theta) = \cos(m\theta)$ . Using Chebyshev polynomials ( $T_m(x) = \cos(m\theta)$  if  $x = \cos\theta$ ) allows to write the Gielis curves in terms of Chebyshev polynomials. With this duality between  $x$  and  $\theta$  Gielis places himself in the sequence Pythagoras-Lamé-Gielis (he had only the polar description of Gielis curves before, but now he also has the Cartesian description).



Rhodonea or Grandi curves

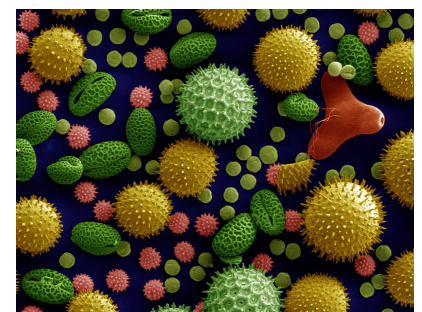


reduced to only two basic shapes: the circle and the (logarithmic) spiral. Applying the Gielis transform to these basic shapes can generate nearly everything.

In the *Demonstratio* plant shapes are tested against the mathematical formulas: plant leaves and stems, tree rings, cell structures, plant pollen,... and their morphogenesis. All these shapes are the result of nature satisfying some optimality condition, possibly using an anisotropic measure.

The *Conclusio* summarises that all is just a generalisation of the Pythagoras theorem and reflects on world views that place plants and flowers, or physics, or humans at the centre.

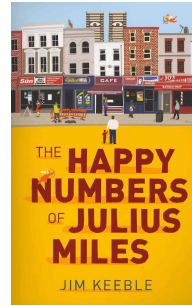
This is a richly illustrated book, based on a simple, but general formula. The applications are found sometimes in an unexpected domain. Gielis was able to link his formula to many very diverse issues in mathematics. It can be seen as an invitation to read his papers that give many more mathematical details. On page 114, the proofreader had a local slump:  $M(\sqrt{2}, 1)$  has an extra bracket and in  $n''' = \sqrt{m''n'}$  the second  $n$  is missing a prime.



plant pollen

**The Happy Numbers of Julius Miles** by *Jim Keeble*. Alma Books, 2012, (283 p.) isbn 978-1-8468-8181-7

The Julius Miles of the title is a mathematician living in a multicultural society of East London. He is a loner, a bit socially defective and shy, but soft hearted, and a very big man: a gentle giant. Being a mathematician means here: obsessed by numbers, a bit autistic, and working as a statistician at the nearby hospital. He lives in a house that he inherited from his uncle Gus, a rich extravert womaniser, operating on the boundaries of the law. Next door he rents out a shop to a 39 year old woman Daisy Perkins.



Jim Keeble

There is some ‘electricity’ between Julius and Daisy, but since she was one of the ‘girls’ of his uncle, he stays away from her except one time when she was behind with the rent. Then there is Larry Silk, an accountant for the criminal network of a ruthless Jay Munnelly. Larry also ‘uses’ Daisy’s shop to whitewash some of the illegally earned money. Some day Julius finds Daisy dead on the floor. Accident or murder remains unclear. She leaves behind a four year old son Arnold who is taken care of by a Somali woman Awa Yasin, who is also looking after her depressed father.

Both Larry and Julius assume they are the father of Arnold and take occasionally care of the boy. So the story starts as a kind of detective thriller. How did she die? What exactly was going on in her shop? If her dead was not an accident, then who murdered her and why? The weirdest character of the novel is Kevin whose father was a policeman who died of cancer. This Kevin had a sex-operation and as a transgender she now calls herself Felicity and she is telling the whole story of the novel. She considers herself as a kind of Cupid or guardian angel, trying to arrange love affairs or to look after people. She is also trying to unravel the true story of what has happened and behaves like a private detective, working on his (or her) own account. She is shadowing people remaining unseen, observing them with binoculars, breaking into houses to get extra information or to listen to conversations, but not intervening and staying out of sight as much as possible. The reader is discovering the events by looking over her shoulder.

So what starts as a thriller with some surprising plot twists, lingers on and becomes a bit of a mealy soft love story and the clumsy characters, unable to utter openly their feelings, are just missing a happy relationship that is meant to be. It is all told in a telegraphic street language ministering the background of the characters in spoonfuls. The narrator Felicity is not the angel that she tries to be. She is addicted to drugs and has police as well as criminal connections. The reader should be flexible to accept less realistic or hard-to-believe events. The story is a bit cartoon-like with extreme characters acting as caricatures.

There are occasionally some mathematics involved. Julius recites in his head all kinds of numbers and statistics when he arrives in a stressful situation. He falls asleep trying to produce an infinite sequence of elliptic curves containing a length-10 arithmetic progression. At the age of 40 Julius and his old school friend re-iterate their previous participation at the Mathematics Olympiad in Cambridge and we are presented four genuine questions (no solutions) they have to solve in three and a half hours. Also the happy numbers of the title refer to a mathematical definition. Take a positive integer and generate a sequence by taking the sum of squares of its digits and iterating this process. If the sequence ends in the number 1, then the original number is called a happy number. Perhaps an unfortunate definition because a happy number will always end in a lonely singular ‘1’. The dramatic end of the novel happens on Arnold’s fourth birthday, and 4 is an unhappy number. Unhappy numbers are not so exceptional. The sequence of unhappy numbers is 2, 3, 4, 5, 6, 8, 9, 11, 12, 14, 15, 16, 17, 18, 20,... (OEIS A031177).

Adhemar Bultheel

Catherine FINET –  
Karl GROSSE-ERDMANN

JOURNEE ORGANISEE AVEC LE SOUTIEN DE  
L'EDT MATH

## Services d'Analyse Mathématique et de Probabilités et Statistique

**10h15**      **Étienne Matheron**  
**(Université d'Artois)**  
*La propriété de Blum-Hanson*

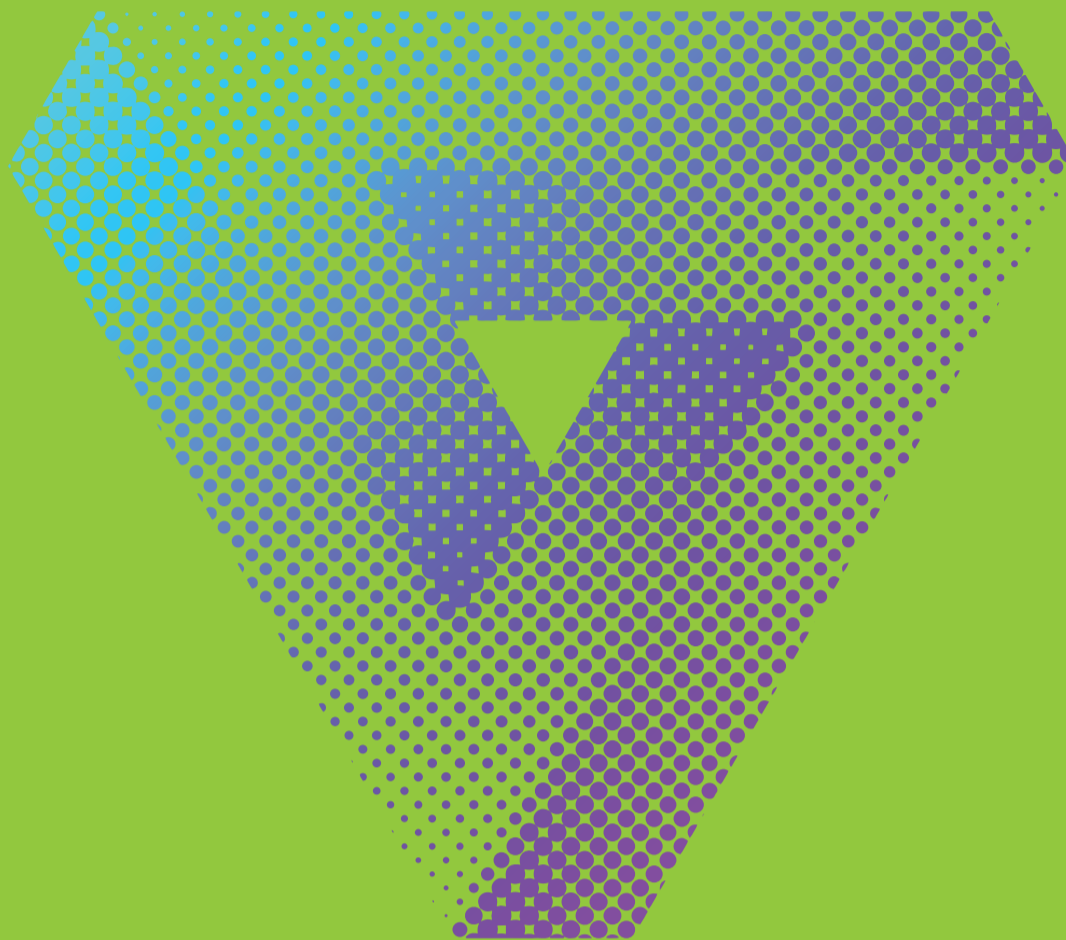
**13h45**      **Sylvie Ruelle**  
**(Université Paris-Sud)**  
*Systèmes dynamiques chaotiques sur  
l'intervalle*

**JEUDI 21 MARS 2019**  
**Le Pentagone – Salle 0A11/rez-de-chaussée**  
**Avenue du Champ de Mars, 6**  
**7000 Mons**

*Invitation cordiale à tous*

# Summer School

## Algebra and Topology



11-14 September 2019

**UCLouvain**

Institut de recherche en mathématique et physique

Silvana Bazzoni

(Università degli Studi di Padova)

**Contramodules and their applications**

Marino Gran

(Université catholique de Louvain)

**Categories for universal algebra**

Sandra Mantovani

(Università degli Studi di Milano)

**Categorical commutator theory**

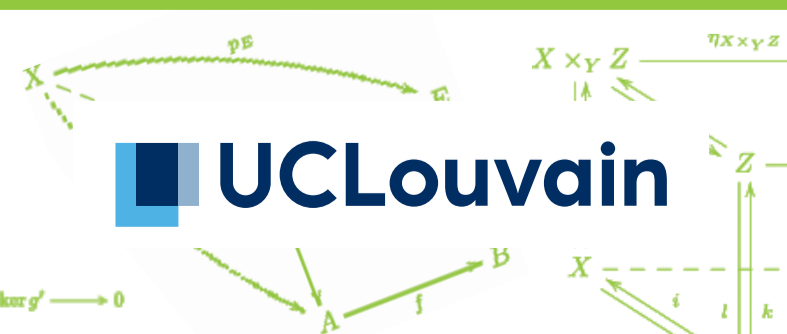
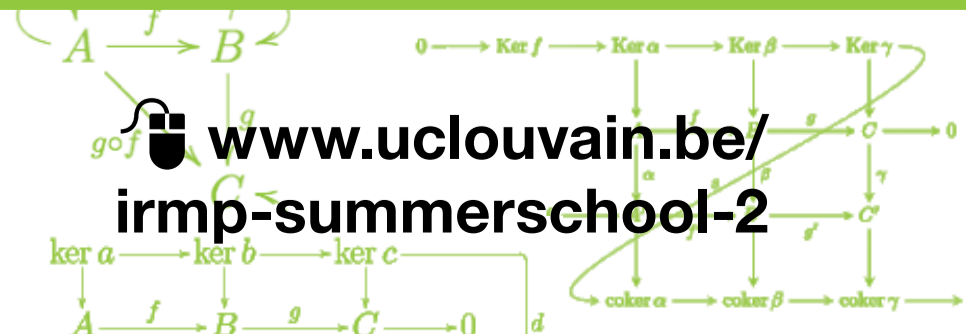
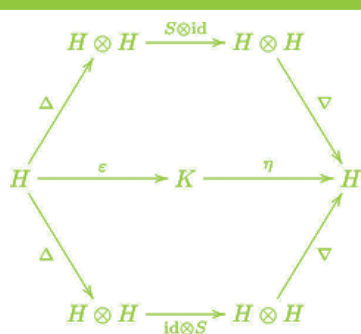
Jorge Picado

(Universidade de Coimbra)

**Frames and locales:  
topology without points**



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