Newsletter

BELGIAN MATHEMATICAL SOCIETY

Comité National de Mathématique CNM

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NCW Nationaal Comité voor Wiskunde

Newsletter of the Belgian Mathematical Society and the National Committee for Mathematics

Belgian Mathematical Society ASBL/VZW
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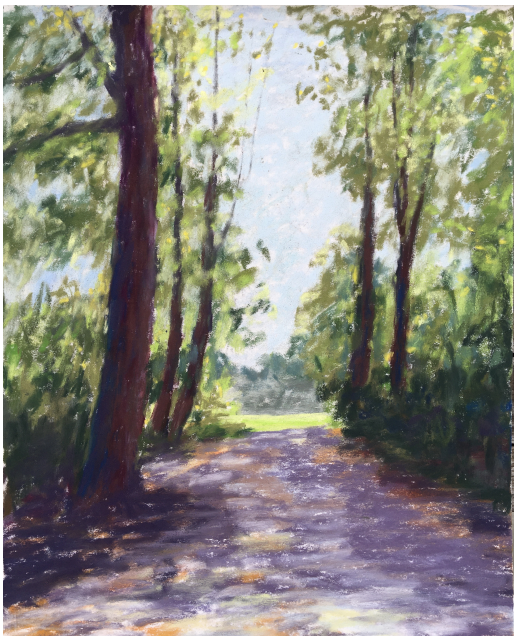
Website: bms.ulb.ac.be
Newsletter: wendy.goemans@kuleuven.be

129, September 15, 2020



Andreas Weiermann deviantart.com beautyandtruth

Contents

The next edition of this newsletter will appear on November 15th, so from now till November 6th all content can be sent to wendy.goemans@kuleuven.be. Any information that you qualify as interesting to be spread among the Belgian Maths community is very much welcomed! Examples of such information are: PhD defenses, seminars, conferences, workshops, meetings, interaction with other sciences or business companies, popular lectures, school initiatives, math exhibitions, job opportunities, . . .

Foreword

Dear BMS members,

This newsletter reaches you as our Society is set to turn centennial during an international crisis which is causing disruptions at all levels of human Society. Like all other professions, ours has also been turned heads over heels. Not only have we had to change our teaching habits, but we have had to rethink the very soul of our craft: the human interactions. Travels have now slowed down to the bare minimum, meetings are cancelled or performed online, squeaky clean Skype/Zoom/Teams sessions have replaced true blackboard discussions with dusty hands and chalked up trousers . . . And none of these musings come close to describing the personal hardships that some of our members have gone through over the last few months.

As every other part of Society, the BMS is also having to rethink the organization of its activities. We will keep you informed of our decisions – which we hope you will appreciate – very soon, in a separate communication.

Until then, we wish you all a successful and happy new academic year, may the current crisis serve you as a stepping-stone for personal and professional development.

At your Service, whatever that may be,

Yours sincerely,
Philippe, Wendy and Yvik

Awards and prizes: Congratulations to all!

Joan Daemen and Vincent Rijmen received RSA award for excellence in mathematics

In February 2020, Joan Daemen and Vincent Rijmen were awarded the annual RSA award for excellence in mathematics, see https://www.rsaconference.com/about/press-releases/rsa-conference-2020-announces-excellence-in-the-field-of-mathematics-award.

Beth prize awarded to Juan Pablo Aguilera (UGent)

Dr. Juan Pablo Aguilera, a member of the research group logic and analysis of the department WE16 of the faculty of science of UGent, won this year the Beth prize for his excellent PhD thesis "Between the Finite and the Infinite" in mathematical logic. The prize comes with a cash award of EUR 3000, a certificate and an invited publication in the FoLLI Publications on Logic, Language and Information published by Springer.

See more about the Beth prize on https://en.wikipedia.org/wiki/E._W._Beth_Dissertation_Prize.

1 News from the BMS & NCM

1.1 Bulletin of the Belgian Mathematical Society - Simon Stevin

In May 2020, Volume 27, Number 1, of the Bulletin of the Belgian Mathematical Society - Simon Stevin appeared with the following table of contents:

- **A. Chapman** Field of Iterated Laurent Series and its Brauer Group. 1–6.
- **W. Śliwa, A. Ziemkowska-Siwek** On block basic sequences in non-Archimedean Köthe spaces. 7–16.
- **F. Daneshvar, A. Razavi** A class of monotonic quantities along the Yamabe flow. 17–27.
- **B. Radičić** The inverse and the Moore-Penrose inverse of a k-circulant matrix with binomial coefficients. 29–42.
- **K. Pryszczepko** The classification of fully filial torsion-free rings. 43–47.
- **X. Zhu** Composition operators and closures of $\mathcal{Q}_K(p,q)$-type spaces in the Logarithmic Bloch space. 49–60.
- **W-F. Xuan, Y-K. Song, W-X. Shi** Generalized Mrówka spaces and diagonal properties. 61–69.
- **G. Amiraliyev, M. Enes Durmaz, M. Kudu** Fitted second order numerical method for a singularly perturbed Fredholm integro-differential equation. 71–88.
- **M. Kubiak, B. Szal** Uniform convergence of trigonometric series with p-bounded variation coefficients. 89–110.
- **M. Novotný** Some Remarks on Schauder Bases in Lipschitz Free Spaces. 111–126.
- **G.N. Ogwo, C. Izuchukwu, K.O. Aremu, O.T. Mewomo** A viscosity iterative algorithm for a family of monotone inclusion problems in an Hadamard space. 127–152.
- **F. Vanhove** The association scheme on the points off a quadric. 153–160.

In July 2020, Volume 27, Number 2, of the Bulletin of the Belgian Mathematical Society - Simon Stevin appeared with the following table of contents:

- **A. Noubissie, A. Togbé, Z. Zhang** On the Exponential Diophantine Equation $(a^n-1)(b^n-1) = x^2$. 161–166.
- **N. Monod** Notes on functions of hyperbolic type. 167–202.
- **T. Henrique de Freitas, V.H.J. Pérez** On Shifted Principles of Generalized Local Cohomology Modules. 203–218.
- **X-S. Peng, Y. Zhang, X. Gao, Y-F. Luo** Left counital Hopf algebras on bi-decorated planar rooted forests and Rota-Baxter systems. 219–243.
- **H-X. Chen, Y. Zhang** Reconstruction of tensor categories from their structure invariants. 245–279.
- **N.D. Verhulst** G-valuations and G-valuation rings. 281–298.
- **J. Zhou** Critical Fujita Exponent for the Porous Medium Equation in $\mathbb{R}^N$ with hole. 299–319.

In August 2020, Volume 27, Number 3, of the Bulletin of the Belgian Mathematical Society - Simon Stevin appeared with the following table of contents:

- **S. Zahiri, A. B. Saeid** Similarity monadic basic logic. 321–336.
- **P. Sosna** Some remarks on phantom categories and motives. 337–352.
- **N. Doğan** Some Remarks on Diametral Dimension and Approximate Diametral Dimension of Certain Nuclear Fréchet Spaces. 353–368.

- **H. Guan, S. Zhou** Classification of point-primitive linear spaces with $2pq$ points. 369–378.
- **O. Dovgoshey** Combinatorial properties of ultrametrics and generalized ultrametrics. 379–417.
- **G. Beer** On boundedly compact metrics and UC metrics. 419–430.
- **H. Xu, J. Sun** Super-biderivations on the $2d$ supersymmetric Galilean conformal algebra. 431–447.
- **H.Y. Xu, X.M. Zheng** Results on the growth of meromorphic solution for a class system of complex functional equations. 449–466.
- **E. Osgooei, A. Fereydooni** Properties of frame mappings devised by controlled p-frames and p-frames. 467–479.

For the table of contents of previous issues, see https://projecteuclid.org/all/euclid.bbms.

Remember, as a member of the BMS you can ask for electronic access to all electronically available issues of the bulletin, if you don't have a login yet, contact pcara@vub.ac.be.

2 (Online) Meetings, Conferences, Lectures, ...

Exposition
Simon Stevin from Bruges (Simon Stevin van Brugghe), 1620 - 2020
He changed the world
August 28 – November 29, 2020

Simon Stevin is one of the most important mathematicians, born in Belgium. This year, we commemorate his death in 1620. He contributed to many areas of science and engineering. He is also very famous because he translated various mathematical terms into Dutch. He introduced the word *wiskunde* for mathematics. The word wiskunde stands for *the knowledge of what is certain*.

He published in 1585 *De Thiende*. Here he extended the positional notation to the use of decimals to represent fractions. The city of Bruges honours him by his statue on the Simon Stevin square in the center of the town.

In Bruges, there is an exposition to commemorate his death in 1620, which can be visited till November 29, 2020. We include here the link from the city of Bruges, and we kindly invite all of you to visit this exposition.

See also https://www.visitbruges.be/en/simon-stevin-1620-2020

References
https://en.wikipedia.org/wiki/De_Thiende
https://en.wikipedia.org/wiki/Simon_Stevin

3 PhD theses

Correspondence theorems in Hopf-Galois theory for separable field extensions

Bui Hoan-Phung
ULB
September 10, 2020

Thesis advisors: Prof. Dr. Joost Vercruysse (ULB), Prof. dr. Gabor Wiese (Université du Luxembourg)

Summary

Galois theory had a major impact on the mathematics of today. Its most important result is the correspondence theorem which is expressed as follows: if L/K is a finite Galois field extension and if $G = \mathrm{Gal}(L/K)$ is its Galois group, then there exists a bijective correspondence between the intermediate fields of L/K and the subgroups of G. Explicitly, if G_0 is a subgroup of G, then we associate to it the set of G_0-invariants L^{G_0} which is an intermediate field of L/K. On the other hand, if L_0 is an intermediate field of L/K, then we associate to it the Galois group $\mathrm{Gal}(L/L_0)$ which is a subgroup of G.

There are many ways to generalize Galois theory, the one we have chosen uses Hopf algebras. The idea, introduced by Chase and Sweedler, is to replace the action of the group G by an action of a Hopf algebra H. Such extensions are called Hopf-Galois.

The first step towards the generalization of the correspondence theorem is due to Chase and Sweedler: if L/K is a Hopf-Galois extension with Hopf algebra H and if H_0 is a Hopf subalgebra of H, then we can construct the set of H_0-invariants L^{H_0} which is an intermediate field of L/K. Unfortunately, unlike the case of Galois extensions, all intermediate fields of L/K do not occur in this way and a characterization of the fields of the form L^{H_0} does not seem to be known.

The goal of this thesis is to generalize the correspondence theorem for finite separable Hopf-Galois extensions. For this purpose, we gave a natural and intrinsic characterization of the intermediate fields of L/K that can be expressed in the form L^{H_0} for some Hopf subalgebra H_0 of H. Thus, we were able to prove a correspondence theorem for Hopf-Galois extensions that is analogous to the one of Galois theory. We have also established, like in Galois theory, a variant of the correspondence theorem for Hopf subalgebras that are normal.

A key component of this thesis is provided by the work of Greither and Pareigis. They have associated a group to a finite separable Hopf-Galois extension. We proved that it was possible to translate the correspondence theorem in terms of this group. Moreover, this group allowed us to build an alternative Hopf-Galois structure on the initial field extension. This helped us to better understand the correspondence theorem.

Finally, we proposed a definition of Hopf-Galois extensions for infinite separable field extensions and obtained very encouraging results. This opens up new possibilities for future research.

Decomposition algebras and axial algebras

Michiel Van Couwenberghe
Ghent University
September 18, 2020, at 16h30
Ghent University, campus Sterre, building S25, auditorium Emmy Noether
Krijgslaan 281, 9000 Gent

Thesis advisors: Prof. dr. Tom De Medts (UGent) and Prof. dr. Hendrik Van Maldeghem (UGent)

Summary

In 1982, Robert L. Griess proved the existence of the Monster group by constructing a 196884-dimensional non-associative algebra over $\mathbb{R}$, called the *Griess algebra*. A peculiar feature of this algebra is the existence of many idempotents with the property that multiplication by each of these idempotents gives rise to a decomposition into eigenspaces. The multiplication of eigenvectors has to obey a very precise *fusion law*.

Alexander A. Ivanov introduced Majorana algebras in order to axiomatize a large class of real non-associative algebras obeying the same fusion law as the Griess algebra. Only recently, in 2015, the more general concept of *axial algebras* was introduced by Jonathan I. Hall, Sergey Shpectorov and Felix Rehren. On the one hand, they are generalizations of Majorana algebras relaxing some of its axioms and defined over an arbitrary field. On the other hand, they can be seen as generalizations of commutative, associative algebras and Jordan algebras as well. The subject has received a lot of attention since then, especially the connection with groups, in the form of *Miyamoto groups* of axial algebras.

It became apparent that there was a need for a more general framework to study axial algebras. In this thesis we introduce (general) *fusion laws* and *decomposition algebras* as natural generalizations of axial algebras. These decomposition algebras admit a natural notion of morphisms, making them into a nice category. We explain how axial decomposition algebras fit into this framework by defining *axial decomposition algebras*. We also provide general constructions to produce examples of (axial) decomposition algebras.

Gradings of fusion laws can now be interpreted as morphisms of fusion laws and lie at the root of the connection with groups. We explain how this connection is not functorial. However, we also introduce a more universal connection (the *universal Miyamoto group*) which turns out to be functorial under some minor conditions.

We define *modules* over (axial) decomposition algebras and vitalize the connection with groups by relating well-behaved modules to representations of the corresponding group. Especially for Matsuo algebras, a type of axial algebras related to 3-transposition groups, this connection is very strong.

A large part of the thesis is devoted to the explicit construction of (axial) decomposition algebras on which the complex *Lie algebras* of simply laced type act by derivations. The Miyamoto group of these algebras is the corresponding Lie group. This class of algebras includes a 3876-dimensional algebra for the Lie algebra of type E_8. The existence of such an algebra was already observed by Skip Garibaldi and Robert M. Guralnick outside the context of axial algebras. However, no explicit construction was known and this has already led to new information and insight into the algebra.

Moment maps in multisymplectic geometry

Leyli Mammadova
KU Leuven
October 1, 2020, 13h00

Thesis advisor: Prof. dr. Marco Zambon (KU Leuven)

Summary

Multisymplectic, or n-plectic, geometry is the generalization of symplectic geometry, the latter corresponding to $n = 1$. Symplectic geometry is the study of smooth manifolds equipped with a closed nondegenerate 2-form. It originated in the Hamiltonian formulation of classical mechanics, where the phase space of a physical system is a symplectic manifold. Multisymplectic geometry arose from the attempts to formulate classical field theory in a similar way, first appearing in the works of W. M. Tulczyjew, J. Kijowski and W. Szczyrba. The *multiphase space* of such a theory is an n-plectic manifold, that is, a smooth manifold equipped with a closed nondegenerate $(n+1)$-form.

The moment map in symplectic geometry relates the Lie algebra of symmetries of a symplectic manifold to the Lie algebra of its smooth functions, called the Lie algebra of *observables*. This notion, introduced by J. M. Souriau, generalized the already known examples of linear and angular momentum from classical physics (hence the name), and turned out to have many applications in both physics and mathematics.

The natural question of generalizing this notion to multisymplectic geometry has led to multiple definitions. This thesis focuses on 2 notions of n-plectic moment maps that generalize other definitions existing in the literature.

The first one is the *homotopy moment map* introduced by M. Callies, Y. Fregier, C. L. Rogers and M. Zambon. Just as the symplectic moment map relates the Lie algebra of symmetries of a symplectic manifold to its Lie algebra of observables, the homotopy moment map relates the Lie algebra of symmetries of an n-plectic manifold to its algebra of observables. The difference is that in the case of an n-plectic manifold, its algebra of observables is not a Lie algebra, but a *Lie n-algebra*.

Another generalization of the symplectic moment map we consider is the *weak moment map* introduced by J. Herman. A homotopy moment map restricted to a certain subspace of its domain is an example of a weak moment map. However, there are situations where a weak moment map exists, but a homotopy moment map does not. The two notions are compared in this thesis; in particular, we investigate when existence of a weak moment map implies existence of a homotopy moment map.

Situations where a homotopy moment map doesn't exist lead to another phenomenon: existence of "homotopy moment maps" for something called *central extensions*[1] of Lie algebras of symmetries. When the manifold under consideration is n-plectic, the corresponding central extension of the Lie algebra of symmetries is a *Lie n-algebra*. Thus, we naturally arrive at the notion of a "homotopy moment map" whose domain of definition is a Lie n-algebra rather than a Lie algebra. This naturally leads us to investigate more general "homotopy moment maps" from Lie n-algebras (i.e., not only the ones that are central extensions of Lie algebras). In this thesis, Lie 2-algebra moment maps are defined, questions of their existence and uniqueness are investigated, and examples are provided.

[1] Such a situation in symplectic geometry corresponds to what is known as "classical anomaly" in mechanics.

Finally, a class of multisymplectic manifolds that, in particular, generalizes coadjoint orbits in symplectic geometry is introduced, and a homotopy moment map is constructed for a subclass of these manifolds.

See also https://agenda.kuleuven.be/nl/node/74610

4 Job announcements

4.1 From KU Leuven

4.1.1 Junior Professor in Zuivere wiskunde (campus Kulak, Kortrijk)

Aan de Groep Wetenschap en Technologie, Faculteit Wetenschappen, Departement Wiskunde van KU Leuven is er een voltijdse vacature in het zelfstandig academisch personeel in het domein van de Zuivere Wiskunde op Campus Kulak in Kortrijk. Op Campus Kulak worden Bachelors in de Wetenschappen onderwezen, met doorstroomopties naar de farmaceutische wetenschappen, de ingenieurs- en de bio-ingenieurswetenschappen. Het is dan ook een erg multidisciplinaire omgeving, zowel voor wat betreft onderwijs als onderzoek. Het wiskundig onderzoek op de campus situeert zich in de onderzoeksgroepen Algebraïsche Topologie en Groepentheorie (zuivere wiskunde) en Wiskundige Natuurkunde (modellering en simulatie van fysische golffenomenen & signaalanalyse) van het Departement Wiskunde.

Meer informatie op https://www.kuleuven.be/personeel/jobsite/jobs/55577262.

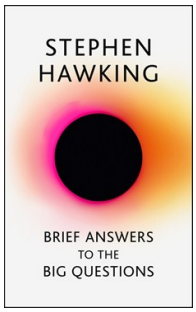
Solliciteren tot 30 september 2020.

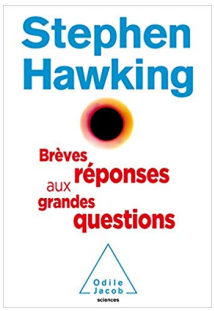
5 History, maths and art, fiction, jokes, quotations ...

5.1 Adhemar's corner

To start the reading list of this academic year, here follow two book reviews by Adhemar. The first, *Stephen Hawking, "Brief Answers to the Big Questions"*, is on the last book of Hawking, published posthumously in which he gives his answers to Big Questions of humanity. Secondly, a thriller with mathematicians and theoretical physicists as the main characters of *Nova Jacobs, "The Last Equation of Isaac Severy"*. Enjoy reading!

Stephen Hawking's last book:
Brief answers to the big questions, John Murray, 2018 (256 p.) isbn: 978-1473695986
De antwoorden op de grote vragen, Spectrum, 2018 (263 p.) isbn: 978-2738145673
Brèves réponses aux grandes questions, Odile Jacob, 2018 (240 p.) isbn: 978-2738145673





During his life Stephen Hawking did a lot of magic with mathematics to derive revolutionary results in cosmology. Together with Penrose about the origin of our universe and later about the physics of black holes and how they could evaporate by radiation. He also showed that thereby information is not annihilated, so that the second law of thermodynamics still holds. Because of his bright mind despite the ASL he became a rock star in a wheelchair using his computerized voice as a trademark. As a famous scientist he was asked for his opinion about the big questions of humanity. During the last year of his life he was working on this book in which he bundles his (extensive) answers to ten of these questions. It was unfinished when he died on 14 March 2018, so his colleagues finalized it.

The book has a foreword by Eddie Redmayne who played the character of Hawking in the movie *The theory of everything* (2014). An introduction is written by his colleague and coauthor Kip S. Thorne, sketching the work by Hawking and how the questions are related to it. Also Hawking himself starts with an autobiographic part about his scientific findings which he uses to explain why it is important to ask these "Big Questions", but the main part of the book consists of his extensive answers. They can be read independently, so there is sometimes a bit of repetition.

• *Is there a God?* Here he leaves the reader free to believe or not in a supernatural being, but for him, the simplest and most feasible conclusion of his research is that there isn't.

• *How did it all begin?* This is related to the previous question, but basically asking the question does not make sense because computing backwards shows that everything began with the singularity of the Big Bang, so there is nothing before it, like there is no point south of the south pole.

• *Is there other intelligent life in the universe?* He is skeptic that life forms similar to ours would exist. There are many reasons why we haven't found any sign of them, but he supports that we should continue looking for them.

• *Can we predict the future?* Determinism according to Laplace in made impossible by the Heisenberg uncertainty relation, and quantum mechanics only can predict probabilities, but evaporating black holes can disperse information, so his answer is most likely no.

• *What is inside a black hole?* He explains how information sucked in by a black hole can be stored on the event horizon. It should tell us what is inside. No information is lost.

• *Is time travel possible?* Most probably not. Not only would it lead to paradoxes, but even though possibilities exist in theory, there is no evidence so far that it would be possible in reality.

• *Will we survive on Earth?* He is rather pessimistic with climate change, pandemics, and available nuclear war missiles with irresponsible political leaders in charge.

• *Should we colonise space?* It will not at all be easy, but it is the only way out when space and resources of our planet are fully consumed.

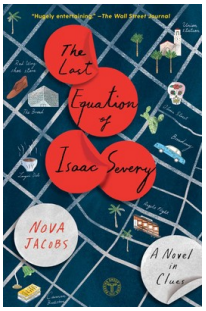
• *Will artificial intelligence outsmart us?* It is almost too late, but we should start seriously worrying about it. Now computers are only faster but soon, they will become smarter than humans.

• *How do we shape the future?* The only way is to invest in good education and promote science.

This is the clearest general introduction to his work and his derived answers to these questions of all times. Written with a positive attitude and with tongue-in-cheek humour. Highly recommended.

Adhemar Bultheel

The last equation of Isaac Severy, *Nova Jacobs*, Simon & Schuster, 2018 (368 p.) isbn: 9781501175138.



Nova Jacobs

Here is one more novel, a thriller this time, in which mathematicians and mathematics plays an essential role. Isaac Severy is a mathematician, specialised in chaos theory at Caltech. He is officially working on an equation to describe the traffic in Los Angeles. He has however secretly worked out some "equation" that can predict place and precise time (date, hour, minute, and seconds) when some homicide or suicide will take place. This is a mathematical version of the TV series *Person of interest* that ran from 2011-2016. However in that series it is a computer (that has some personality) that collects and analyses all the data from cameras and mobile phones to predict where someone is in danger.

Here it is a less convincing equation that predicts it. Jacobs is wise enough not to go into a theory of nonlinear differential or difference equations but she stays at the surface about what the equation even vaguely looks like, except that it is very long and complicated. If such an equation fell into the hands of criminals or even the government, it would be dangerous since they would be able to perfectly predict the future. In fact Isaac has hired a secret hotel room where he left a computer and a map with dots indicating the place and time of the unnatural deaths predicted by his model. This includes his own death (presumably a totally unexpected and hence suspicious suicide). Isaac dies in the preface of the book and it is the MacGuffin of the whole novel. His granddaughter Hazel receives a letter from him in which she is asked to save the equation and give it only to a mysterious person she does not know and keep everything very secret. The letter has also a hint to the hidden hotel room 137 she has to unravel.

The actual novel feeds the reader with clues about where this equation is and what it is about (and about several other complications). And Hazel is not the only one who is looking for it. Here it requires to understand the rather complicated relation between the many (perhaps too many) weird characters, which is a bit difficult to remember at first.

<table>
<tr><td colspan="6">Isaac × Lily</td></tr>
<tr><td colspan="3">Philip × Jane</td><td>Paige</td><td colspan="2">Tom × Carla</td></tr>
<tr><td>Sybil × Jack</td><td>Silas</td><td>Sidney</td><td>Alex</td><td>Hazel</td><td>Gregory × Goldie</td></tr>
<tr><td>Drew</td><td colspan="3"></td><td colspan="2">Lewis</td></tr>
</table>

First there is Isaac (the grandfather). His wife Lily, a major in both comparative literature and mathematics, is now demented and stays in an institution. There are three children all geniuses. The oldest son Philip is also a professor at Caltech. He is a theoretical physicist specialized in string theory, but his scientific output is somewhat stagnating. As a youngster he has solved one of the lesser Hilbert's problems, but some other mathematician had it published a few weeks before him. Philip is married to Jane (a molecular biologist at Stanford and Harvard) and they have a daughter Sybil (an artist and a parasomniac) besides a twin Silas and Sidney who are tennis champions. Sybil is married to Jack and they also have a daughter Drew (still a preschooler) that is some kind of an autistic savant.

Isaac has also a daughter Paige who is a brilliant statistician (working on a book for decades and that she will probably never finish). She is very much on her own and assumed to suffer from

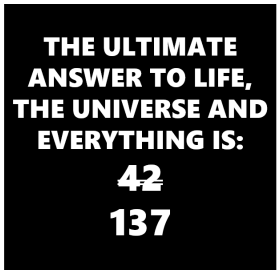
alexithymia (the inability to identify and describe emotions in her self). She has never married, but she has a son Alex (what's in a name), Alex also is a prodigy who won a fellowship for the Max Planck institute in Bonn, but after a car accident dropped out and stayed in Europe working as a freelance photographer.

Tom, Isaac's youngest son, married to Carla. Like Philip he suffers from migraine, but he takes his resort to street drugs. Both Tom and Carla are junkies. Nevertheless, they adopted two orphans: Hazel and her brother Gregory (an obvious reference to Hansel and Gretel). Hazel has a book store and Gregory is a police officer at LAPD. Tom and Carla neglect and even maltreat the children and when Carla dies and Tom is arrested, Hazel and Gregory were adopted by Isaac and Tom becomes a persona non grata of the family. Hazel recovers rather well as an adult, but Gregory keeps an aggressive killer's instinct, and there are some other family members that have hidden darker sides. It takes a while before all these family ties and all their psychology becomes clear to the reader. The chapters are told from the viewpoints of Hazel, Gregory and Philip.

The whole clan flocks together for the funeral of Isaac. In Isaac's cryptic letter to Hazel it is also predicted that three will die and he is the first. Indeed two others of this family will die (of course not the ones that the reader expects). Eventually several people find out about this equation, which turns out to be a fake because it contains an error and it will be Hazel (the only one that is not a genial scientist) that will find the correct formula and it is suggested that she eventually will be converted to a mathematician. There are some additional charecters that are not family members, and some of them are also after the equation. The most important is Nellie Stone (representing her virtual boss P. Booth Lyons), from a company that operates as a matchmaker between science and government. Of course there are also some illegitimate love affairs: Philip with a bright Ukrainian PhD student Anitka Durov, and Gregory with Sybil.

I do not want to include spoilers by disclosing the whodunits or who dies how and where. The nice thing about the book is not only that many of these characters are mathematical geniuses but also that there are many references to issues from mathematics and physics. Let me give a few examples.

The logo of the organization of Booth Lyons is a Fibonacci spiral, which is described as a brain-like doodle. Haze overhears a heated discussion about Riemann's zeta function and the sense or nonsense of mathematical abstraction. Except for Newton's *Principia Mathematica*, the books that Hazel fetches from Isaac's library seem all to be fictional and so are of course also the lectures about branes and M-theory that Philip is preparing. Isaac explains to Hazel and Gregory when they were still young the principles of chaos theory by telling that one may predict the next random drip from a leaky kitchen faucet but that the slightest perturbation may change the outcome completely. Of course Lorentz's butterfly effect is also mentioned elsewhere.

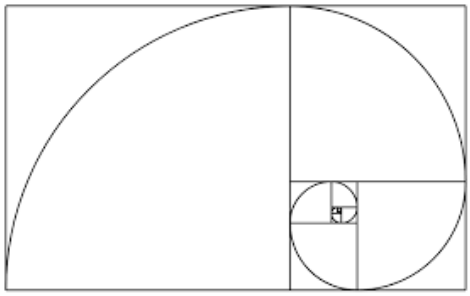
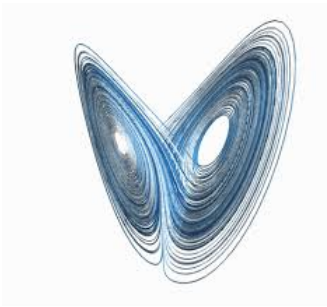
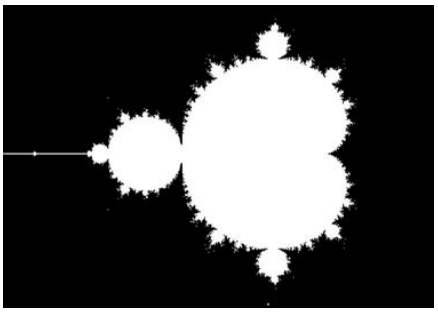


The last, yet to be discovered element with atomic number 137.

$1729 = 1^3 + 12^3 = 9^3 + 10^3$

Many physicists and mathematicians are mentioned and discussed in different contexts: Oppenheimer, Einstein, Schrödinger, Heisenberg, Turing, and Weyl, even their assumed love life. John Britton however, a monster mind from Princeton, is fictional. Philip for example hopes his picture will once be between two other very real Nobel Prize winners from Caltech: Richard Feynman and Marray Gell-Mann. Hazel's initials H.S. are not mistaken by Alex for Horst Störmer, another Nobel Prize winner.

The number 137 is not only prime but it also plays an important

role in physics (and also in the novel). Wolfgang Pauli was obsessed by it and he died in a hospital room with that number. It is strange that Anitka claims that the search for the "God particle" has failed to produce measurable experimental results. However the Higgs-Englert particle was detected in 2012. This can not be explained by the time frame of the novel which plays at the end of 2015. Isaac for example died at 101715055531, which is 17 October 2015 at 5:55:31 AM.


Pierre de Fermat

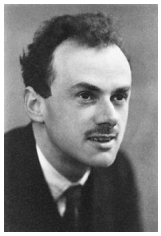
Fermat Numbers

$F_n = 2^{2^n} + 1$

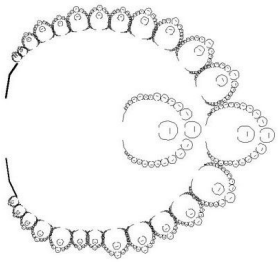
Fermat Primes

$F_0 = 2^{2^0} + 1 = 3$
$F_1 = 2^{2^1} + 1 = 5$
$F_2 = 2^{2^2} + 1 = 17$
$F_3 = 2^{2^3} + 1 = 257$
$F_4 = 2^{2^4} + 1 = 65537$

When looking for a password, Axel tries many mathematical constants (Euler's constant, Planck's constant, e, the golden ratio, Ramanujan's number (1729), Mersenne primes, the largest known prime ($2^{57885161} - 1$, although a larger one $2^{82589933} - 1$ was found in 2018)). The number π is not among them, probably considered to be too obvious as a password. Someone falls down a stair with 257 steps, which is identified as a Fermat prime but Philip refers to the numerology of searching a number with some meaning after every event as blatant nonsense. Drew (as a preschooler) can recite all the first 100 prime numbers. There is a poem that is often attributed to Paul Dirac that he wrote in his twenties:



Age is of course a fever chill
That every physicist must fear
He is better dead than living still
When past his thirtieth year


Fractal nature of an African village

There is a discussion about fractals and about the self-similarity in the fractal shapes of African villages when Nellie serves Philip a cappuccino in which the foam is representing a Mandelbrot set.

Jacobs plays an interesting shell game with her readers, setting them on the wrong tracks and delivering clues bit by bit. This is culminating in thrilling events when, after the interpretation of the numbers on the map has become clear to some of the characters, the readers witness how the tension is whipped up as the fatal moment is approaching. But there is an happy ending. The good guys turn out to be not so good but the bad ones are not so bad after all, while the survivers live happily ever after. The reader however is left uninformed about the true nature of Isaac Severy's last equation, which will not be a surprise if it is a mathematician. Adhemar Bultheel