Newsletter

BELGIAN MATHEMATICAL SOCIETY

135, November 15, 2021

Comité National de Mathématique CNM

NCW Nationaal Comité voor Wiskunde

Newsletter of the Belgian Mathematical Society and the National Committee for Mathematics

Belgian Mathematical Society ASBL/VZW
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The next edition of this newsletter will appear on January 15th, hence, till January 8th all content can be sent to wendy.goemans@kuleuven.be. Any information that you qualify as interesting to be spread among the Belgian Maths community is very much welcomed! Examples of such information are: PhD defenses, seminars, conferences, workshops, meetings, interaction with other sciences or business companies, popular lectures, school initiatives, math exhibitions, job opportunities, ...

Foreword

Dear BMS members,

We here at the BMS board hope that you are all thriving and enjoying a successful first half of the academic year. This Newsletter's main point is to invite you all to attend our General Assembly and conference to be held, as already announced, on Wednesday December 15th. All the necessary information is available below. We hope many of you will be able to attend in person!

All the best,
Philippe, Wendy and Yvik

1 News from the BMS & NCM

1.1 Recent breakthroughs in Mathematics and General Assembly 2021

The Belgian Mathematical Society is happy to invite you to its "Recent breakthroughs in Mathematics and General Assembly 2021" which will take place Wednesday 15/12/2021 from 2PM to 6PM.

Aside from the yearly general assembly of the BMS, the event will as usual consist in an afternoon's discussion by international experts on some of the more breathtaking breakthroughs in contemporary mathematics.

This year the talks will focus on topics which were recently awarded prestigious prizes (particularly the Abel prize and the Shaw prize). All talks will be accessible to large audiences of mathematicians.

On this occasion, the BMS will also award its first "Young Scholar Award", about which more information is available on the dedicated website.

The program:

- 14h00-14h20: Coffee
- 14h20-14h30: Welcome address by Yvik Swan (president of the BMS)
- 14h30-15h20: Shaw prize laureate Jean-Michel BISMUT (Orsay)
- 15h30-16h20: Abel prize laureate Avi WIDGERSON (Princeton) - online talk
- 16h30-17h10: Coffee break + BMS General Assembly
- 17h10-18h00: Professor Imre BARANY (Alfréd Rényi Mathematical Institute)

The meeting will be closed by a cocktail offered by the BMS to all participants.

The event will take place in auditorium B1.315 in building B on Campus Solbosch of ULB, Brussels. Due to the limited available seating, and also to possible COVID restrictions, we will ensure that the event can also be followed online; specific information on this option will be made available in due course.

Participation is free but registration is mandatory, see all information on

http://dwispc8.vub.ac.be/nieuwBMS/index.php?id=recent-breakthroughs-and-general-assembly-2021

1.2 Bulletin of the Belgian Mathematical Society - Simon Stevin

In May 2021, Volume 28, Number 1, of the Bulletin of the Belgian Mathematical Society - Simon Stevin appeared with the following table of contents:

- **José Bonet, Werner J. Ricker** Fréchet and (LB) sequence spaces induced by dual Banach spaces of discrete Cesàro spaces
- **Stanisław Sędziwy** Boundary value problems with singular $\varphi-$Laplacians
- **Ruishen Qian, Xiangling Zhu** A new characterization of certain Dirichlet type spaces with application
- **Baghdadi Aloui, Jihad Souissi** Jacobi polynomials and some connection formulas in terms of the action of linear differential operators
- **Laiachi El Kaoutit, José Gómez-Torrecillas** On the finite dual of a cocommutative Hopf algebroid. Application to linear differential matrix equations and Picard-Vessiot theory.
- **V.V. Tkachuk** A note on $\kappa-$Fréchet–Urysohn property in function spaces
- **Dongwei Guo, Wenchang Chu** Binomial Sums with Pell and Lucas Polynomials
- **Tom Kaiser** A closer look at the non-Hopfianness of $BS(2,3)$

Starting from Volume 28 the Bulletin of the Belgian Mathematical Society - Simon Stevin only appears online and is not printed any more. As a member of the BMS you have electronic access to all electronically available issues of the bulletin, free of charge. You have received instructions for this by e-mail. If you have any trouble logging in or accessing the journal, please contact

customer_support@projecteuclid.org.

For the table of contents of previous issues, see https://projecteuclid.org/all/euclid.bbms.

1.3 Membership dues for 2022

The basic BMS membership fee is 20€ per year. See Section 1.3.1 for reciprocity membership.

You can either pay via bank transfer (**BIC: GEBABEBB / IBAN: BE70 0011 7447 8525**)
or via PayPal (see http://bms.ulb.ac.be/membership/paypal.php).

Our address is:

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Campus de la Plaine, C.P. 218/01
Boulevard du Triomphe
B-1050 Brussels, BELGIUM
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1.3.1 Reciprocity and combined membership

The BMS has reciprocity agreements with the AMS, EMS, DMV, LMS, RSME, SMF, SBPMef, VVWL and KWG. In case you are already member of one of these societies, your membership fee for the BMS is reduced to 18€. Details can be found on this webpage.

We summarize the most common combined memberships:

BMS	20,00€
BMS with reciprocity	18,00€
BMS + EMS	45,00€
BMS + EMS with reciprocity	43,00€

Note that the EMS (European Mathematical Society) membership fee of 25,00€ is allowed only to persons belonging to an EMS corporate member society, such as the BMS. The individual EMS membership fee is 50,00€ otherwise.

Note that it is now preferred that you pay your EMS membership fee directly to the EMS.
See http://www.euro-math-soc.eu/ems_payment_new/ems_payment_new.html for details.

2 Announcements

2.1 Shaw Prize Award Ceremony 2021

The virtual Shaw Prize Award Ceremony took place in October (see https://www.shawprize.org/) and at that occasion also the Laureates in Mathematical Sciences Jean-Michel Bismut (Université Paris-Saclay) and Jeff Cheeger (New York University) were honoured.

The BMS is honoured to welcome Jean-Michel Bismut on the occasion of its Recent breakthroughs in Mathematics event, see the announcement earlier in this newsletter!

2.2 Ingrid Daubechies in New York Times Magazine

The New York Times Magazine published recently an article devoted to Ingrid Daubechies, title "'The Godmother of the Digital Image"':

https://www.nytimes.com/2021/09/14/magazine/ingrid-daubechies.html

3 (Online) Meetings, Conferences, Lectures, ...

3.1 November 2021

Visit and seminar in Mathematics

As part of the EUNICE alliance, Jesse Beisegel will visit the mathematics department of UMONS on November 23, 24 and 25 and will give the following talk:

Convex Structures in Graphs - Jesse Beisegel
(BTU Cottbus-Senftenberg - EUNICE)
November 23, 11.00-12.00
Pentagone 0A07, Av. du Champ de Mars 17 - 7000 Mons

Two-day Workshop on Singularities in Geometry and Dynamical Systems

November 22-23, 2021 (Hasselt)

This is a joint event organised by the dynamical systems group in Hasselt (headed by Peter De Maesschalck), the geometry group at KU Leuven (headed by Joeri Van der Veken and Marco Zambon), and the analysis group in Antwerpen (headed by Sonja Hohloch). Its aim is to bring together young researchers working on one of these subjects and provide gentle introductions to topics they may not be familiar with.

The event with consist of

- 1-hour minicourses by each of the four researchers mentioned above and by external speakers Marie-Amélie Lawn (Imperial College London) and Ale Jan Homburg (Korteweg-de Vries Institute & VU Amsterdam)
- 8 short presentations by postdocs and PhD students.

Registration is free but compulsory, by November 18. Detailed information can be found at

https://www.uantwerpen.be/nl/personeel/sonja-hohloch/private-webpage/conference-workshop/minicourse-on-singul/

3.2 Spring 2022

Mini-course "Trusses: between braces and rings"

April - May 2022 (ULB)

Professor Tomasz Brzeziński (University of Swansea, United Kingdom, and University of Białystok, Poland) was awarded an 'international chair' at the *Université Libre de Bruxelles* (ULB). He will visit the department of Mathematics of the ULB between April 24 and May 22, and during this stay he will teach an 8h mini-course addressed to Master and PhD students and any one else with an interest in (new) algebraic structures.

A (provisional) schedule and abstract of the course is given below.

- Thursday 26 April, 14h-16h30
- Thursday 5 May, 14h-16h30
- Thursday 12 May, 14h-16h30
- Thursday 19 May, 14h-16h30

All lectures will take place in the 'Salle Solvay' at ULB, Campus Plaine, building NO, 5th floor.

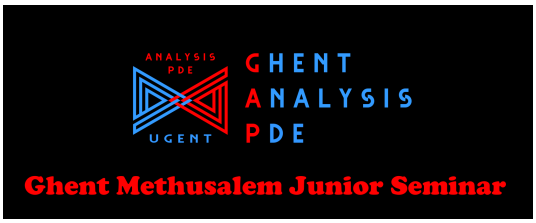
"Trusses: between braces and rings"

An algebraic structure is a collection of sets with operations. Typical and most widespread across mathematics are systems such as a semigroup, monoid, group, ring, field, associative algebra, vector space or module. In this lecture course we will study some simple algebraic systems which have recently gained prominent position in algebra and topology such as braces, racks or quandles (sets with two operations interacting with each other in prescribed ways). In particular we will explore a

little known fact (first described nearly 100 years ago by Pruefer and Baer) that one can give a definition of a group without requesting existence of the neutral element and inverses by using a ternary rather than a binary operation (i.e. an operation with three rather than the usual two inputs). A set with such a suitable ternary operation is known as a heap. By picking an element in a heap, the ternary operation is reduced to the binary group operation, for which the chosen element is the neutral element (the resulting group is known as a retract). We will study properties and examples of heaps and relate them to the properties of corresponding groups (retracts). Next we will look at heaps with an additional binary operation that distributes over the ternary heap operation, known as trusses, relate them to both rings and braces, and study their properties and applications.

3.3 Seminars and colloquia

Ghent Methusalem Junior Seminar



The Ghent Methusalem Junior Seminar is run by PhD students and postdocs at the **Ghent Analysis & PDE Center** (https://analysis-pde.org).

It provides an ideal opportunity for young researchers in mathematics to share their ideas and to learn about new trends in a wide range of fields. Targeting a mainly (though not exclusively) young audience has meant for the organizers to ensure a relaxed atmosphere and to encourage the audience to engage in stimulating discussions with the speakers, ideally leading to new collaborations.

The seminar currently takes place every Tuesday at 4.30 PM (CET) on ZOOM. For more information about our activity and about past and future talks, please visit the dedicated webpage: https://analysis-pde.org/ghent-methusalem-junior-seminar/

If you would like to give a talk or to invite someone to give a talk, please contact:

- Duvan Cardona Sanchez (Duvan.CardonaSanchez@UGent.be)
- Serena Federico (Serena.Federico@UGent.be)
- Vishvesh Kumar (Vishvesh.Kumar@UGent.be)
- David Rottensteiner (David.Rottensteiner@UGent.be)
- Bolys Sabitbek (b.sabitbek@qmul.ac.uk).

Scheduled talks are (to be updated):

- 16 November 2021, Julio Delgado (Universidad del Valle, Colombia).
- 23 November 2021, Soledad Villar (Johns Hopkins University, USA).
- 30 November 2021, Federico Santagati (Politecnico di Torino, Italy).
- 7 December 2021, Hardy Chan (ETH Zürich, Switzerland).
- 14 December 2021, Elisa Affili (University of Deusto, Spain).

The Ghent Methusalem Junior Seminar is supported by FWO Odysseus 1 Project: Analysis and Partial Differential Equations, and by the Ghent University Methusalem Programme "Analysis & PDE".

Some pictures of Ghent and of our Seminars

Methusalem Colloquium talks

KU Leuven

Scheduled talks for the fall semester are

Thursday 25 November, 4-5 pm in 200K.00.06
Speaker: **Daniel Drimbe (KU Leuven)**
Title: **Cocycle and orbit equivalence superrigidity for group actions**
Abstract: The classification of probability measure preserving actions up to orbit equivalence has witnessed an explosion of activity in the last two decades. In particular, there have been found numerous classes of group actions that are orbit equivalent superrigid, i.e. the actions are completely recovered

from their orbit equivalence relation. The goal of this talk is to survey these results and show that many of them follow from a cocycle superrigidity theorem.

Tuesday 7 December, 4-5pm in 200K.00.06
Speaker: **Aldo Witte (KU Leuven)**
Title: **Adding and removing singularities in symplectic geometry**
Abstract: Hamiltonian mechanics can be mathematically described using a symplectic form; a closed non-degenerate two form on a manifold. But there are instances where symplectic forms are too rigid, and one has to allow for singularities. One class of singularities which is studied in the literature (motivated by generalized complex geometry) are of elliptic type along an embedded codimension-two submanifold; these are forms which are the real part of a holomorphic form with simple poles along the submanifold. But from an algebra-geometric point of view it would be much more natural to allow for simple poles along a non-embedded submanifold. In this talk we will explain how to construct these, and show that in four dimensions these singularities can always be resolved back to occur along an embedded submanifold. Joint work with Gil Cavalcanti and Ralph Klaasse.

See also https://wis.kuleuven.be/methusalem-pure-math/colloquia_seminars_lectures

4 PhD theses

Bridging exceptional geometries and algebras using inner ideals of Lie algebras

Jeroen Meulewaeter
Ghent University

December 10, 2021, 4pm, Auditorium Emmy Noether, S25, Krijgslaan 281, 9000 Ghent (This timing is tentative, the final time will be published on this webpage in two weeks.)

Thesis advisors: Prof. Dr. Tom De Medts (Ghent University) and Prof. Dr. Hendrik Van Maldeghem (Ghent University)

Summary:

Central in this thesis is the concept of an inner ideal in a Lie algebra L, this is a subspace I such that $[I,[I,L]] \leq I$. The 1-dimensional inner ideals are also called extremal points. We use these inner ideals to give constructions of (exceptional) spherical buildings and are particularly interested in the cases of rank 1 and 2, more precisely in the Moufang polygons and Moufang sets.

In a foundational paper in 2006, Cohen and Ivanyos introduced the concept of an extremal geometry in a Lie algebra. This is an incidence geometry with as points the extremal points and certain 2-dimensional subspaces as lines. They show that, if there are lines in this geometry, that it forms a so-called root filtration space and in a later paper they classify these spaces. We generalize this concept of an extremal geometry in order to deal with the case that there are points but no lines in the extremal geometry, and show that this new geometry either coincides with the extremal geometry, forms a polar space, or is just a set without lines.

Structurable algebras are a class of non-associative algebras introduced by Allison that generalize the Jordan algebras. The Tits-Kantor-Koecher (TKK) construction of Jordan algebras can be generalized to structurable algebras. We construct Moufang sets, Moufang triangles and Moufang hexagons using inner ideals of Lie algebras obtained from (specific classes of) structurable algebras via the TKK-construction.

Then we investigate the relation between simple Lie algebras generated by extremal points and various algebraic structures. If the characteristic is not 2 or 3 we show that if such a Lie algebra is not symplectic, it is obtained by applying the TKK-construction to a skew-dimension one structurable algebra. Then we drop this assumption on the characteristic and show that a certain 5-grading related to extremal points is algebraic, which essentially means that for every element in the 1-part of this grading, there exist automorphisms which behave well with respect to this 5-grading. Using these automorphisms, we can recover a (cubic) Jordan algebra if there are lines in the extremal geometry. Then we exploit that correspondence to show that a finite-dimensional simple Lie algebra generated by its extremal points is characterized by its extremal geometry if this geometry has lines. If the extremal geometry contains so-called symplectic pairs, we recover, under some additional assumptions, a quadrangular algebra which allows us to analyse the case of a Moufang quadrangle in detail.

5 History, maths and art, fiction, jokes, quotations ...

5.1 Projet radio sur les maths

Qu'est-ce que le langage mathématique? Quelle est la vie au quotidien des mathématicien.nes? Comment entendre, ressentir les mathématiques? Dans cette création radiophonique, nous explorons l'acte créatif en mathématiques en donnant la parole à des mathématiciennes actuelles et en imaginant différents aspects de la vie de grandes scientifiques méconnues du grand public. A travers des nouvelles, des interviews, et des jeux sonores, nous emmenons l'auditeur.rice aux portes de ce monde trop souvent considéré comme inaccessible.

La création était programmée à Paris le 8 octobre pour une journée Arts et Sciences:
https://iscpif.fr/artex/
Mais sa véritable présentation se fera dans une soirée d'écoute au centre culturel Jacques Franck à Saint-Gilles le 5 décembre.

5.2 $+-=\div\times$ Tour

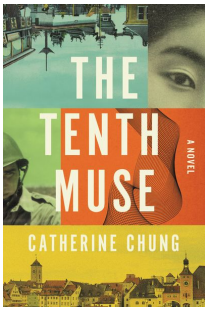
Next year, Ed Sheeran, the English singer-songwriter, will be in Belgium with his "The Mathematics Tour". After his studio albums called $+$ (pronounced "plus"), $\times$ (pronounced "multiply"), $\div$ (pronounced "divide"), the album $=$ (pronounced "equals") appeared. Tickets for the $+-=\div\times$ Tour of Ed Sheeran are still available at https://www.ticketmaster.be.

5.3 Adhemar's corner

This edition contains three reviews by Adhemar. The first is on a novel about a female mathematician standing her ground in a men's world, namely, Catherine Chung's *The tenth muse*. The second is on a dialogue between a teacher and a student who is exploring rational approximations of square root 2, discovering continued fractions, iterations, and its irrationality. All to be found in *The square root of 2* of Richard Flannery. Finally, *How round is a cube?* of James Tanton is an entertaining collection of mathematical problems and puzzles.

Many thanks to Adhemar and enjoy reading these!

The tenth muse, *Catherine Chung*, HarperCollins/Ecco, 2019 (304 p.) isbn: 978-0-0625-7406-0



Catherine Chung

Here we have a novel in which the main character is a mathematician. It is important to distinguish Catheringe Chung, the author, and Katherine, the main character of the novel. There are some similarities, but it is a common mistake to identify an author with a character from his or her imagination. Chung is born in the US but has the looks of a half-white half-Chinese. She grew up partially in Michigan, has a degree in mathematics, but her passion is writing, and she got a Master of Fine Arts from Cornell University and has been a successful author so far. The fiction is the story of Katherine, half-white half-Chinese, growing up in the 1940-50's in Michigan. She is a mathematical prodigy in elementary class, able to sum the numbers 1 to 9 instantly using the Gauss trick by adding the symmetric numbers in the sequence. She thinks this is so obvious that there is no need to write down anything. Instead of being praised, she is punished for her arrogance. Moreover she and her parents are looked down upon for being not a pure traditional white family. But that kind of misery goes on throughout the book, and it is mainly the women that are being maltreated by men. Without any warning mother is leaving and replaced by a stepmother, with whom Katherine has not a good relation. When she goes to the university, she is betrayed by her best friend. Without being asked, she is automatically accused of copying his homework while it was the other way around. When she starts to work on a PhD, she has a crush on her supervisor. When their joint work, which is basically hers, is published, it is the supervisor who gets all the credit for it.

Here the title and the prologue of the novel becomes important. The tenth muse is the one that wants to sing her own song, unlike her nine sisters who are just inspiring others (almost always men). As a consequence all her powers are taken away from her and she has to become human, and she is reincarnating as all the women who throughout history have refused to be the servants of men, and stood their ground to excel in whatever they are good at, and we know some: Hypatia, Sophie Germain, Sofia Kovalevskaya, Mary Mayer, Emmy Noether,...

Meanwhile Katherine finds out that her parents are not her parents, and that she is Jewish and was born probably in Germany. That on top of her being half-breed, trying to make a career in mathematics, a world dominated by men, are a lot of stereotypes all culminating in one character and she goes through some identity crisis. She wants to do research on her own, research that is not being imposed upon her by the supervisor. She starts for a solution of the (fictional) Mohanty problem (In real life Chandra Mohanty is a professor at Syracuse University defending transnational feminism). Solving this would be a major breakthrough to the proof of the Riemann conjecture. She thinks that the (fictional) Shieling-Meisenbach theorem can help her in solving the problem. That theorem was produced by mathematicians from Göttingen around 1940 when she was born. So she grabs the occasion to spend some time in Bonn to work on her own and visit Göttingen. She meets Meisenbach who has known her parents but she is also betrayed by him while her supervisor, without consulting her, finishes her PhD work, this is the icing on the cake of male humiliation. We learn at the end that Katherine has left the topic and became successful in the field of chaos and dynamical systems which has many applications in diverse applied sciences.

Besides this story line, there is a lot of mathematics in the novel, since the decor is mathematics. Chung has borrowed material that can be found in many books on popular mathematics. There are short notes about Gödel, Ramanujan, Turing, Noether, and 30 more real mathematicians. There is the faith of math in Göttingen in the 1940's, the Riemann hypothesis, the zeta function, the Boltzmann equation, but all without technicalities. We learn about the ratrace of being the first to publish a result, the importance of collaboration and yet being cautious not to give away too much. And above all: its a men's world!

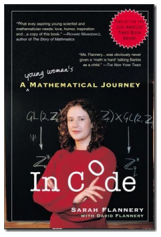
Adhemar Bultheel

The Square Root of 2: A dialogue concerning a number and a sequence, Praxis Publishing, 2006 (256 p.) by *David Flannery*

David Flannery

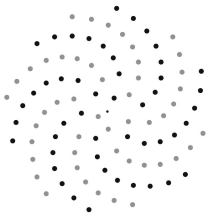
The author David Flannery is an Irish mathematician, who used to tease his eldest daughter Sarah with puzzles and he coached her in solving mathematical problems that she loved doing. So she made a science project on cryptography as a teenager, by which she won in 1999 first a local competition, then on a national level and eventually she won the internationa *Young Scientist Award*. This was interesting enough for the media to pick up the story. Shortly after that she related her story in a book *In code* that was published in 2001. It was not only biographical, but it also told about her love for mathematics and her dedication to solving problems. Some puzzles were included for amusement of the reader and it became a popular math book that had some international success.

Sarah was the main author of *In Code*, but David certainly has helped. It might have been because of the success of that book that David engaged on the project of writing the present book about the square root of two. It is another math book for a broad audience that is written in a classic style of dialogues. Dialogues between master and student is a technique of teaching that goes back to an antique Greek tradition, and that has been also used by Galileo. For Galileo, it was more a way of letting his characters formulate provocative statements about the heliocentric system that would have been dangerous for him if he had formulated them more directly. In *The square root of 2* we follow the chatty conversation between two people in a master-student relation where the teacher is coaching the student towards discovering some mathematics about numbers and sequences, and the student is clearly finding joy in that. This is most probably inspired by how he has coached Sarah when she was young.



Sarah & D. Flannery

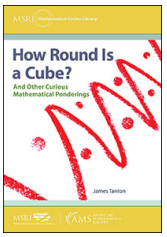
Drawing the diagonal of a square with side 1 gives $\sqrt{2}$, but what is this number? First rational approximations are tried. Integer solutions to the Pell equations $m^2 = 2n^2 \pm 1$ give rational approximations m/n, in fact a sequence of alternating upper and lower bounds $\frac{1}{1}$, $\frac{3}{2}$, $\frac{7}{5}$, $\frac{17}{12}$, $\frac{41}{29}$, $\frac{99}{70}$, $\ldots$; $\frac{m}{n} \to \frac{m+2n}{m+n}$ involving the Fibonacci sequence and recurrence. This gradually leads to continued fractions and the Heron sequence $a \to \frac{1}{2}(a + \frac{2}{a})$ to approximate $\sqrt{2}$ which is actually Newton's method. Grouping several iteration steps into one gives convergence acceleration methods. The above sequence is also the sequence of best approximations: $\frac{m}{n}$ is a better approximation than any rational approximant with a smaller denominator. Here Flannery introduces some notes on Ramanujan and Gauss (both were good at spotting patterns). Summing formulas and formulas for triangular numbers are derived. Irrationality is illustrated graphically by drawing dots at the points given in polar coordinates by $r = \sqrt{n}$ and $\theta = n\phi\pi$ with $\phi = \sqrt{2}$ resulting in spirals. This holds for any other irrational number ϕ, for example the golden ratio or its inverse showing the spiral patterns seen in sunflowers.

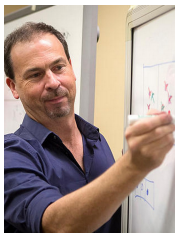


There are a few notes with additional literature or historical remarks, like that the notation $\sqrt{\ }$ was introduced by Christoff Rudolff in 1525 and derived from the letter 'r' for radix. The book makes some interesting links between simple mathematical elements from number theory, but everything remains at a really low level. For example it is explained that $\neq$ means not equal and $\approx$ means approximately equal. In my opinion the book is somewhat long winding and after a while it is really tempting to leaf through or skip a number of pages. If could be used as a guideline if the reader wants to coach a promising young pupil, or it could be a read assignment to be explored by a young teenager with some interest in mathematics.

Adhemar Bultheel

How round is a cube? And other curious mathematical ponderings, MSRI-AMS, 2019 (xiv+262 p.)
by *James Tanton*



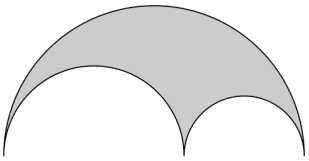


James Tanton

James Tanton is a mathematician whose main activity is related to raising mathematical awareness for a general public and helping teachers and students in secondary schools by providing joyful mathematical problems. He writes books, has a website (*jamestanton.com*), he tweets, posts videos, gives lectures, etc.

This book has 34 chapters that collect mathematical problems and puzzles and their solution and each chapter has one or more "research topics" that coud be used for further exploration, either individually or in class. So, it is a collection of mainly curious fun problems, the ones that are usually not on the school curriculum, but it is more than just fun because some mathematical rigour is maintained and the research challenges are very instructive. The usual aversion that is triggered by an overload of abstraction, technicalities, and formulas, is avoided a much as possible. There are formulas, but the many colourful figures make most of them unnecessary. Though the graphics, the proofs look like almost obvious. This influences the selection of problems because they often have some "geometric flavour" even though the original problem does not seem to have any geometric interpretation.

Examples of geometric problems are related to the classical arbelos. The figure was studied by Archimedes from Syracuse and one of the problems is to prove that the area of the arbelos equals the area of the circle whose diameter is the vertical common tangent to the small semi-circles inside the big one. This vertical divides the arbelos in two parts. The Archimedean circles are the largest ones that fit inside each part and Archmedes proved that these are equal, which is a less simple problem. Many other problems can be invented just based on three circles, For example the curve described by the centers of the circles that are tangent to two of the three semi-cicles generate conics.



arbelos

Another geometric problem is Feynman's triangle: Connect every vertex with a point on the opposite side cutting it in a ratio $1/3$. What is the area of the inner triangle defined by these lines with respect to the original one? What if the ratio is $1/r$ instead or $1/3$? How about squares instead of circles?

The title of the book is also the title of a chapter on 3D geometric objects. Consider an arbitrary polyhedron. How spherical is it? For this one has to study circles whose center is on a vertex of solids with flat faces. A flat circle spans an arc of 360°. At the vertex of a cube such a circle consists of three, and not four quarter circles. For a tetrahedron, this is three arcs of 60°, etc. In general we get for vertex v $(1 - x_v)360°$ and x is called the "pointiness" of that vertex. The "pointiness" of a polyhedron can then be defined as $\sum_v x_v$, a sum over all vertices. Step by step, the reader is led to the Euler characteristic $V - E + F$ where $(V, E, F) = \#$(vertices, edges, faces).

But there are also many less geometric examples involving Ramsey theory, L^p metrics, binary numbers, combinatorics, number theory, graph theory,.... However for all of these, there is some graphical representation, even if it is not really a geometrical problem. What I mean by this is best illustrated with the number theoretical problems. The Pythagorean triples form an excellent example. Similarly for the chapter on trapizoidal numbers (the number of nodes in a regular trapizoidal grid) or the chapter about triangular numbers with some side conditions. It is about numbers, but there is a geometry of triangles and squares behind.

Constructions on grids are used a number of times. There is a chapter on grid polygons, which are polygons whose vertices are on the nodes of a regular square grid. This is introduced as Gauss' shoelace problem. It was indeed Gauss who discovered a formula for the area of such a polygon given the (integer) coordinates of its vertices. Another one in this vein is to construct equilateral grid polygons, and to compute the area of grid triangles, which is given by Pick's theorem (by counting the number of gridpoints inside or on the boundary). There is also a chapter on the Prouhet-Thue-Morse sequence, with original proofs of its recursive properties, one of which is based on folding paper with red and blue sides in a particular way, rater than construction a binary sequence based on the binary representation of integers. Some problems are more demanding than other, but there are a lot of fun ones to be found.

Adhemar Bultheel