BELGIAN MATHEMATICAL SOCIETY
\# 139, September 15, 2022

Comité National de Mathématique CNM


Newsletter of the Belgian Mathematical Society and the National Committee for Mathematics

Belgian Mathematical Society ASBL/VZW
ULB Campus Plaine, C.P. 218/01, Bld du Triomphe, B-1050 Brussels, Belgium
Website: bms.ulb.ac.be
Newsletter: wendy.goemans@kuleuven.be


By Andreas Weiermann

## Contents

1 News from the BMS \& NCM ..... 2
2 (Online) Meetings, Conferences, Lectures,... ..... 3
3 PhD theses ..... 12
4 History, maths and art, fiction, jokes, quotations ... ..... 14

The next edition of this newsletter will appear on November 15th, hence, till November 9th all content can be sent to wendy.goemans@kuleuven.be.

## Foreword

Dear BMS members,
On behalf of the BMS board, we wish you all a fruitful, cheerful, and joyous new academic year full of happy encounters, delighted students, and wonderful theorems.

We hope to meet you during one of our events, for instance during our commemoration of Jacques Tits which is planned in December 2022 at the Académie.

All the best,
Céline, Joost, Wendy and Yvik

## 1 News from the BMS \& NCM

### 1.1 Algebras, Geometries, and Groups

A conference in memory of Jacques Tits (1930-2021)

When: Tuesday 20th December and Wednesday 21th December 2022
Where: Académie Royale de Belgique (Rooms Albert II and Salle de marbre) Rue Ducale 1, Bruxelles
Registration: Attendance is open to everyone, but registration is compulsory, see below for registration (no google address is required; if you have any trouble with the registration form then just send an email to yvik(dot)swan(at)ulb(dot)be).
Registration fee: If you register (and pay the registration fee) before October 15th: attendance is free for PhD students and 30 EUR per person per day for non-PhD students. From October 16 onwards, attendance is 30 EUR per day for PhD students and 50 EUR per person per day for non- PhD students. After registration, you will receive an email confirmation along with information about the method of payment. Registration includes lunches and coffee breaks.

## Confirmed Speakers:

- Jean-Pierre BOURGUIGNON (IHES): Reminiscences about Science and Jacques Tits
- Michel BROUE (Paris)
- Bernhard MÜHLHERR (Gießen): Buildings and Root Gradings
- Anne PARREAU (Grenoble)
- Guy ROUSSEAU (Nancy)
- Katrin TENT (Münster): Sharply 2-transitive groups and the Burnside problem
- Donna TESTERMAN (Lausanne): Overgroups of regular unipotent elements, finite and algebraic
- Jean-Pierre TIGNOL (Louvain-La-Neuve)
- Richard WEISS (Tufts): Tits Polygons

Organisers: Pierre-Emmanuel CAPRACE (UCLouvain), Ann DOOMS (VUB), Fabien DURAND (UPJV, SMF), Simone GUTT (ULB), Alain VALETTE (UNeuchatel, SMS), Hendrik VAN MALDEGHEM (UGent), Joost VERCRUYSSE (ULB, BMS).

With the administrative support of the Belgian Mathematical Society via Céline ESSER (ULiège) and Yvik SWAN (ULB).

Sponsors: FWO Scientific Research Network "Coding Theory and Cryptography".
For the most recent information and registration, see http://dwispc8.vub.ac.be/nieuwBMS/index.php?id=agg

### 1.2 Bulletin of the Belgian Mathematical Society - Simon Stevin

Starting from Volume 28 the Bulletin of the Belgian Mathematical Society - Simon Stevin only appears online and is not printed any more. As a member of the BMS you have electronic access to all electronically available issues of the bulletin, free of charge. If you have any trouble logging in or accessing the journal, please contact customer_support@projecteuclid.org.

For the table of contents of previous issues, see https:/ / projecteuclid.org/all/euclid.bbms.

## 2 (Online) Meetings, Conferences, Lectures, ...

### 2.1 September 2022

## Chaire de la Vallée Poussin

Since its creation in 2001, the "Chaire de la Vallée Poussin" is awarded every year at the Université catholique de Louvain to a world-renowned mathematician (https://uclouvain.be/en/research-institutes/irmp/chaire-de-la-vallee-poussin.html) . It is held in the form of a 4-day-series of lectures with international public at the Institut de Recherche en Mathématique et Physique. An inaugural lecture on the first day is followed by 3 more lectures, exploring the topic in some more depth.

The recipient of the Chaire de la Vallée Poussin 2022 is Ieke Moerdijk, professor at Utrecht University (https://uclouvain.be/en/research-institutes/irmp/news/chaire-de-la-vallee-poussin-2022.html).

Ieke Moerdijk made many important discoveries and contributions in various fields of mathematics, including algebraic topology, category theory, logic and differential geometry. His various research articles and books are well-known worlwide and have influenced the work of many mathematicians during the last decades.

Professor Moerdijk is a member of the Royal Netherlands Academy of Arts and Sciences and of Academia Europeae, and was awarded the Spinoza Prize in 2012.

The Inaugural Lecture entitled "Some interactions between Algebra and Geometry" took place on Tuesday, 6 September 2022 (https:/ /uclouvain.be/fr/instituts-recherche/irmp).

## 9th Heidelberg Laureate Forum

During this one-of-a-kind event, 200 selected young researchers in mathematics and computer science will spend a week of scientific exchange with the recipients of their disciplines' most prestigious prizes.

All important information on the 9th HLF at a glance, as well as how to follow it remotely, right here: https://www.heidelberg-laureate-forum.org/

# The Beautiful Impact of Mathematics in Society Wiskunde en Data Science voor Onderwijs, Industrie en Maatschappij 

September 21 2022, VUB

Op woensdag 21 september organiseert de vakgroep Wiskunde en Data Science van de VUB de vijfde editie van het congres "The Beautiful Impact of Mathematics in Society" met als thema "Wiskunde en Data Science voor Onderwijs, Industrie en Maatschappij".

Wiskunde speelt een steeds belangrijkere rol in alle lagen van de maatschappij. Ingenieurs en wetenschappers gebruiken wiskunde, maar wist je dat wiskunde aan de basis ligt van niet minder dan 2.300.000 jobs in Nederland, wat overeenkomt met 30\% van het BNP van Nederland? (https:/ / platform wiskunde.nl/rapport-deloitte/). Toch moeten we niet alleen naar deze cijfers kijken om de impact en het belang van wiskunde te begrijpen. De wiskunde, in samenspel met haar toepassingsgebieden, is in staat om de maatschappij fundamenteel te veranderen! Dat illustreren we op 21 september 2022 opnieuw door sprekers uit de bedrijfswereld, de universiteit én het onderwijs aan het woord te laten.

- prof. dr. ir. Liesbet Geris (KUL en ULg) zal uitleggen of wiskundige modellen patiënten overbodig maken in de geneeskunde.
- dr. Tim Hellemans (ORTEC) zal illustreren dat data voor de maatschappij en industrie een steeds belangrijker rol spelen en wat je er wiskundig allemaal mee kan doen.
- dr. An De Wispelaere (Conundra) zal tonen hoe wiskunde aan de basis ligt van efficiënte algoritmen en hun toepassingen in een bedrijfskundige context.
- dr. Valery Vermeulen (wiskundige, data scientist, muzikant) zal ons tonen (en laten horen) dat wiskunde ook een vorm van kunst is. Hij levert o.a. een muzikale interpretatie van data afkomstig uit het universum.

Van het hoger onderwijs wordt verwacht dat het afgestudeerden aflevert die breed inzetbaar zijn. Gelukkig kan je in België nog steeds afstuderen als wiskundige, waardoor je aan dat profiel voldoet. Aan het secundair onderwijs worden eveneens grote verwachtingen gesteld. Vaardigheden zoals digitale geletterdheid zijn geen onredelijke eis als je denkt aan de evolutie van onze maatschappij. Daarom laten we ook dr. Didier Deses (VUB, wiskundige, leerkracht secundair onderwijs) aan het woord. Hij zal het hebben over verantwoorde digitalisering in het wiskunde-onderwijs. Tenslotte stellen we ook onze eigen vernieuwde opleiding Wiskunde en Data Science voor.

Zal je binnenkort afstuderen als wiskundige? Of als ingenieur of wetenschapper met een zeer sterke wiskundige basis? Ben je geïnteresseerd om uit data inzichten te distilleren? Ben je op zoek naar een (eerste) job in Data? Ook dan ben je bij ons op 21 september aan het juiste adres! Als extraatje laten we immers het bedrijf Ormittalent aan het woord over traineeships. Je leest er meer over op https://ormittalent.be/en/traineeships/data-chief-traineeship/ én je kan tijdens de lunchpauze informeel verder praten met vertegenwoordigers van dit bedrijf! Daarenboven hebben we ook andere
bedrijven uitgenodigd die afgestudeerde wiskundigen aanwerven! Voor studenten en jong afgestudeerden een opportuniteit, voor leerkrachten en leerlingen een prima illustratie hoe snel je als wiskundige kan doorgroeien op de arbeidsmarkt.

Deze vijfde editie van BIMS wordt georganiseerd op de Brussels Humanities, Sciences \& Engineering Campus (campus Etterbeek-Elsene) van de Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussel, in auditorium QA. Het volledige dagprogramma en alle informatie vind je online:
https://we.vub.ac.be/en/beautiful-impact-mathematics-society-bims-v
Deelname aan dit congres kan na registratie en betaling van 25 euro. In deze prijs is een lunch inbegrepen tijdens de middagpauze en u kan verder ook nog kennismaken met HeLiWi (https:/ /heliwi. com). Tijdens de registratieprocedure kan $u$ ook aanduiden of $u$ een attest van navorming wenst. Leerkrachten kunnen hun leerlingen meebrengen, leerlingen nemen gratis deel! We houden minstens 200 plaatsen vrij voor leerlingen. Op dit ogenblik zijn reeds 116 plaatsen gereserveerd voor leerlingen. Tijdens de registratieprocedure kan $u$ aangeven of u graag leerlingen meebrengt, en wordt u gevraagd om een schatting op te geven van het aantal. Wie leerlingen wenst mee te brengen, zal in september gevraagd worden om het precieze aantal op te geven. Van zodra het gereserveerde aantal plaatsen ongeveer bereikt wordt, is het mogelijk dat we de registratie voor leerlingen moeten afsluiten. Registratie is mogelijk tot en met 14 september via de website.

## International Conference: Dynamical Systems, Modeling, and Mathematical Sciences

September 23-25 2022, Dubai/UAE

The aim of the conference is to bring together all researchers in every discipline of applied mathematics, science, as well as engineering, industry to discuss the advances on the dynamical systems and the developments on new mathematical models, theories, and applications that contribute to the advancement of scientific knowledge and practice.

All information is here: https:/ /dynamicalsystemsandmodeling.wordpress.com/

### 2.2 October 2022

## ICHPS 2022: 16. International Conference on Hilbert Problems and their Solutions

October 13-14 2022, London, United Kingdom

International Conference on Hilbert Problems and their Solutions aims to bring together leading academic scientists, researchers and research scholars to exchange and share their experiences and research results on all aspects of Hilbert Problems and their Solutions. It also provides a premier interdisciplinary platform for researchers, practitioners and educators to present and discuss the most recent innovations, trends, and concerns as well as practical challenges encountered and solutions adopted in the fields of Hilbert Problems and their Solutions.

All information is here: https:/ /waset.org/hilbert-problems-and-their-solutions-conference-in-october-2022-in-london

# International Conference: Multidisciplinary Aspects in Mathematics and its Applications (ICMAM 2022) 

October 24-28 2022

Jointly organised by Universidad de Pamplona (Colombia), Universidad de Sao Paulo (Brazil), Freie Universität Berlin (Germany), and Ghent Analysis \& PDE Centre, UGent, Belgium.


## Confirmed Plenary talks

- Terence Tao, UCLA, USA.
- Michael Ruzhansky, Ghent University and Queen Mary University of London, Belgium and UK.
- Tatiana Toro, Department of Mathematics, University of Washington, USA. Vice President International Mathematical Union.
- Simon Donaldson, Imperial College London, UK.
- Carlos Kenig, University of Chicago. President International Mathematical Union.
- David Dos Santos Ferreira, Institut Élie Cartan, Université de Lorraine, France.
- Enrique Zuazua, Deusto University, Friedrich-Alexander-Universität Erlangen-Nürnberg, Universidad Autónoma de Madrid, Germany and Spain.


## Description

The Department of Mathematics at the Universidad del Valle, Cali-Colombia, is delighted to invite you to the International conference: Multidisciplinary Aspects in Mathematics and its applications (ICMAM) 2022, Latin America. The honoree of this year at the conference is the Colombian Mathematician José Raúl Quintero, 2011 National Mathematics Award, Colombian Mathematical Society (Universidad del Valle, Cali-Colombia).

The International conference: Multidisciplinary Aspects in Mathematics and its applications (ICMAM) seeks to contribute to the development of mathematical research in Latin America and the Caribbean, stimulate its visibility and promote exchange between mathematicians of the region and from other parts of the world. The conference will be a video conference and will take place on the 25th-28th of October, 2022

Official website https:/ / sites.google.com/view/matematicasunivalleicmam2022/home

## Organisers

- Chair/President ICMAM 2022: Prof. Dr. Brian Grajales Triana, (Universidad de Pamplona, Colombia).
- Co-chair: Karina Navarro Gonzalez (Universidad de São Paulo, Brazil).
- Milton Manuel Aguirre (Universidad São Paulo, Brazil).
- Jessica Gonzalez Hurtado (Freie Universität Berlin, Germany).
- Julio Delgado, (Universidad del Valle Cali, Colombia).
- Marlio Paredes, (Director of the Graduate Program in Mathematical Science at Universidad del Valle, Cali-Colombia).
- Hector Jairo Martínez, (Chair of the Department of Mathematics at Universidad del Valle, CaliColombia).
- President of the Scientific Committee: Duvan Cardona Sanchez (Universiteit Gent, Belgium).


## Scientific Board

- Duvan Cardona Sanchez, President of the Scientific Board, Ghent University, Belgium.
- Emanuel Carneiro, ICTP, The Abdus Salam International Centre for Theoretical Physics.
- Alicia Dickenstein, Former Vice-President, International Mathematical Union, University of Buenos Aires, Argentina.
- Uwe Kaehler, President of the ISAAC, International Society for Analysis, its Applications and Computations, and University of Aveiro, Portugal.
- Alf Onshuus, President of the SCM, Colombian Mathematical Society, Universidad de Los Andes, Colombia.
- Claudia Garetto, Queen Mary University of London United Kingdom.
- Mitsuru Sugimoto, Member of the Mathematical Society of Japan and Nagoya University Japan.
- Thaís Jordão, University of São Paulo, Brazil.


## Confirmed Speakers

- Tohru Ozawa, Waseda University, Japan.
- Kristin E. Lauter Meta / Facebook.
- Manuel Del Pino, University of Bath, UK
- Andrés Villaveces Universidad Nacional de Colombia.
- Boris Zilber Oxford, UK.
- Thaís Jordão, University of São Paulo, Brazil.


## Registration

https://docs.google.com/forms/d/e/1FAIpQLSeDgsBaIMH95PLKcf6WhLXsOAhABAq1lob5L_X_J5_8nok-9A/viewform


## Workshop on Additive Combinatorics and Algebraic Connections

October 24-28 2022

This workshop brings together researchers with diverse backgrounds ranging from algebra and algebraic geometry to combinatorics, harmonic analysis, and theoretical computer science. The aim is to learn from each other's results and methods in topics of common interest.

All information is here: https://www.ias.edu/math/events/workshop-additive-combinatorics-and-algebraic-connections

### 2.3 November 2022

# International Conference on Applied Science Mathematics and Statistics (ICASMS) 

November 2 2022, Glasgow, United Kingdom

The ICASMS conference is an international forum for the presentation of technological advances and research results in the fields of Applied Science Mathematics and Statistics. The conference will bring together leading researchers, engineers and scientists in the domain of interest from around the world. We warmly welcome previous and prospected authors submit your new research papers to ICASMS, and share the valuable experiences with the scientist and scholars around the world.

All information is here: http:/ /scienceplus.us/Conference/20878/ICASMS/

## 2022 Field of Dreams Conference

November 4-5 2022

The Field of Dreams Conference introduces potential graduate students to graduate programs in the mathematical sciences at Alliance schools as well as professional opportunities in these fields. Scholars spend time with faculty mentors from the Alliance schools, get advice on graduate school applications, and attend seminars on graduate school preparation and expectations as well as career seminars.

All information is here: https:/ /www.ima.umn.edu/2022-2023/SW11.4-5.22\#

## International Conference on Mathematics, Statistics and Applied Science (ICMSA)

November 222022

An elegant and rich premier global platform for the International Conference on Mathematics, Statistics and Applied Science ICMSA that uniquely describes the Academic research and development across globe. This event tries to fill in the void that is being created by the current global academic and research by the different global challenges.

All information is here: http:/ /sciencefora.org/Conference/14271/ICMSA/

The key intention of ICASMS is to provide opportunity for the global participants to share their ideas and experience in person with their peers expected to join from different parts on the world. In addition this gathering will help the delegates to establish research or business relations as well as to find international linkage for future collaborations in their career path. We hope that ICASMS outcome will lead to significant contributions to the knowledge base in these up-to-date scientific fields in scope.

All information is here: http://issrd.org/Conference/14153/ICASMS/

### 2.4 June 2023

## Conference on 100 Years of Noetherian Rings

June 19-23 2022

In the early 1920s, Emmy Noether introduced the fundamental concept of a Noetherian ring, a notion that has had a remarkably broad impact on mathematics over the last century. In this conference, we celebrate Noether's legacy with research talks from many areas of algebra, broadly construed, including algebraic geometry, commutative algebra, number theory, and representation theory.

All information is here: https:/ /www.ias.edu/math/events/conference-100-years-noetherian-rings

### 2.5 Seminars and colloquia

## First Minicourse Event of the Excellence of Science Project "Beyond Symplectic Geometry" October 10 2022, ULB

See more information on
https://www.uantwerpen.be/nl/personeel/sonja-hohloch/private-webpage/excellence2-of-scien/minicourses/


#### Abstract

Altaïr

Le Centre d'Histoire des Sciences et des Techniques reconnu par le Conseil de la Recherche de l'ULB présente le programmes des conferences de l'année academique 2022-2023.


See all information at the end of this newsletter.

## Analysis \& Geometry Seminar <br> UAntwerpen (usually Wednesdays 16-17h during term)

This is the weekly research seminar of the analysis \& geometry-interested people in Antwerp. During the semester, we have once per week a research talk in analysis and/or geometry and/or related
topics. The list of speakers comprises researchers from Antwerp as well as other universities. Details (schedule, speakers, titles, abstracts, seminar room/ online/ hybrid etc.) can be found on the seminar webpage
https://www.uantwerpen.be/nl/personeel/sonja-hohloch/private-webpage/seminars/analysis—geometry-/
To be added/deleted from the mailing list, please send an email to: sonja dot hohloch AT uantwerpen dot be

## Ghent Geometric Analysis Seminar



The Ghent Geometric Analysis seminar is dedicated to studying the modern techniques of elliptic and subelliptic partial differential equations (PDEs) that are used to establish new results in differential geometry and differential topology. We are planning to invite several of the leaders in the fields of microlocal analysis, geometric analysis, and harmonic analysis abroad.

In view of the recent activities and investigations undertaken by the members of the Ghent Analysis and PDE center and the works in the interplay of geometric analysis and harmonic analysis of our group, our seminar also will be a scenario for presenting the recent developments in the field and their applications to other branches in mathematics. Visit the website of our new Ghent Geometric Analysis Seminar at https:/ /analysis-pde.org/seminars/ghent-on-geometric-analysis/

Scheduled talks are (to be updated):

- 24 October 2022, Uwe Kähler, PDE, University of Aveiro, Portugal.
- Durvudkhan Suragan, Analysis and PDE, Nazarbayev University, Kazakhstan, (TBA).

Organisers:

- Duvan Cardona Sanchez (Duvan.CardonaSanchez@UGent.be)
- David Santiago Gómez Cóbos (davidsantiago.gomezcobos@ugent.be)
- Andrea Vanessa Hurtado Quiceno (andreavanessa.hurtadoquiceno@ugent.be)

Visit also the website of the seminar to be informed of the scheduled intensive minicourses about geometric analysis.

## Ghent Methusalem Junior Seminar

The Ghent Methusalem Junior Seminar is run by PhD students and postdocs at the Ghent Analysis \& PDE Center (https:/ / analysis-pde.org).


It provides an ideal opportunity for young researchers in mathematics to share their ideas and to learn about new trends in a wide range of fields. Targeting a mainly (though not exclusively) young audience has meant for the organizers to ensure a relaxed atmosphere and to encourage the audience to engage in stimulating discussions with the speakers, ideally leading to new collaborations.

The seminar currently takes place every Tuesday at 4.30 PM (CET) on ZOOM. For more information about our activity and about past and future talks, please visit the dedicated webpage: https:/ /analysis-pde.org/ghent-methusalem-junior-seminar/

If you would like to give a talk or to invite someone to give a talk, please contact:

- Duvan Cardona Sanchez, Ghent University, (Duvan.CardonaSanchez@UGent.be)
- Serena Federico, Università di Bologna, (serena.federico2@unibo.it)
- Vishvesh Kumar, Ghent University, (Vishvesh.Kumar@UGent.be)
- David Rottensteiner, Ghent University, (David.Rottensteiner@UGent.be)
- Bolys Sabitbek, Queen Mary University of London, (b.sabitbek@qmul.ac.uk)

Scheduled talks are:

- 5 October 2022, Yu Deng (University of Southern California, US).
- 12 October 2022, Cristiana de Filippis (University of Parma, Italy).
- 19 October 2022, Minhyun Kim (Universität Bielefeld, Germany). 26 October 2022.
- 26 October 2022, Jean-Daniel Jjida, (Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany).
- 23 November 2022, Nicolas Camps (Université Paris-Sud, France).
- (TBA), Arick Shao (Queen Mary University of London, UK).


## Ghent Methusalem Colloquium



The Ghent Methusalem Colloquium is intended for a broad audience of PhD students, postdocs and professors at the Ghent Analysis \& PDE Center and beyond. The series includes colloquia from visiting and invited guests. Visit the website of our new Ghent Methusalem Colloquium at https://analysis-pde.org/ghent-methusalem-colloquium/

Visit the webpage of the colloquium to have a look of the scheduled talks by:

- Prof. Andreas Seeger, University of Wisconsin-Madison, US.
- Prof. Roland Duduchava, University of Georgia Tbilisi, Georgia.
- Prof. Eugene Shargorodsky, King's College London, UK.
- Prof. Durvudkhan Suragan, Navarbayev University, Kazakhstan.
- Prof. Julio Delgado, Universidad Del Valle, Cali-Colombia.

The Ghent Methusalem Junior Seminar and the Ghent Methusalem Colloquium are supported by FWO Odysseus 1 Project: Analysis and Partial Differential Equations, and by the Ghent University Methusalem Programme "Analysis \& PDE".


## 3 PhD theses

# Zeros of matrix-valued orthogonal polynomials and critical measures on compact Riemann surfaces 

Alan Groot<br>KU Leuven<br>May 4, 2022

Thesis advisors: Prof. Dr. Kuijlaars (KU Leuven)
Summary:
This thesis mainly concerns non-Hermitian scalar-valued or matrix-valued orthogonal polynomials because of their connection to hexagon tiling models. In the case of doubly periodic tilings of the hexagon, matrix-valued orthogonal polynomials play a fundamental role. First, we study a concrete example of matrix-valued orthogonal polynomials that play a role in a family of $1 \times 2$-periodic tilings of the hexagon. We show that the matrix-valued orthogonal polynomials can explicitly be expressed in terms of scalar orthogonal polynomials. This allows us to express the asymptotic zero distribution of the determinant of the matrix-valued orthogonal polynomials in terms of those of the asymptotic zero distribution of the scalar orthogonal polynomials. Moreover, under a certain geometrical condition, we are able to determine the asymptotic zero distribution of the upper-right entry of the matrix-valued orthogonal polynomial.

Second, we extend the notion of critical measures, critical sets and $S$-curves in the complex plane to compact Riemann surfaces of genus $g \geq 1$ and we use a max-min energy method to prove the existence of critical sets. Moreover, in case $g=1$, we show that critical measures are supported on trajectories of quadratic differentials. In this case, a so-called ( $2,-1$ )-Cauchy kernel plays an important role.

Finally, we study a family of orthogonal polynomials associated to the normal matrix model which satisfies a planar Hermitian orthogonality. This planar Hermitian orthogonality can be converted to a non-Hermitian orthogonality on a contour, which allows for a Riemann-Hilbert analysis to obtain the strong asymptotics.

# Cohomological invariants of structurable algebras 

Simon Rigby<br>Ghent University<br>16 September 2022, 16:00, room 3.1, building S1, Krijgslaan 281, 9000 Gent

Thesis advisors: Prof. dr. Tom De Medts (UGent) and Prof. dr. Hendrik Van Maldeghem (UGent)
Summary:
Jacques Tits published two prescient papers in 1966 that had a role in this PhD. In Classification of algebraic semisimple groups, he wrote down what became known as Tits's Witt-type Theorem. Echoing Witt's Theorem on quadratic forms, it says that a simple algebraic group is uniquely determined by an index (the Tits index, or Satake diagram), and a subgroup called the anisotropic kernel. In Algèbres alternatives, algèbres de Jordan et algèbres de Lie exceptionelles, I. Construction, he devised a way of constructing exceptional Lie algebras, and the result became known as his (and Freudenthal's) magic square.

If the Lie algebra of a simple algebraic group has a $\mathbb{Z}$-grading then the Lie algebra of its anisotropic kernel is in the 0-component. In many interesting cases this $\mathbb{Z}$-graded Lie algebra has just three or five nonzero components. Bruce Allison in 1978 had the technical vision to put a nonassociative algebra structure on the 1-component from which the whole Lie algebra can be reconstructed. He called this type of algebra structurable, axiomatised it, and classified all the central simple structurable algebras in characteristic 0 . The tensor product of two composition algebras is one such algebra, and this is the one responsible, by way of speaking, for Tits's magic square construction.

The PhD thesis is about exceptional structurable algebras (the ones that turn up in exceptional Lie algebras) from the perspective of Galois cohomology. The idea of cohomological invariants, which came from the mind of Jean-Pierre Serre, is to map objects of a certain kind, like algebras or quadratic forms, to elements of Galois cohomology groups in a natural way.

Among other results, we found three cohomological invariants of bioctonion algebras (i.e., the tensor product of two octonion algebras) and two cohomological invariants of Brown algebras (an exceptional 56 -dimensional structurable algebra). The situation with bioctonion algebras is beautiful because the invariants, taking values in $H^{d}(F, \mathbb{Z} / 2 \mathbb{Z})$ for $d=1,3,6$, are a complete list, and they carry information about the algebras. The degree 1 and 3 invariants are related by an elegant formula to the Rost invariant of $E_{8}$, which has values in $H^{3}\left(F, \mu_{60}^{\otimes 2}\right)$, while the degree 6 invariant can be used to define very highdegree invariants, with values in $H^{6}(F, \mathbb{Z} / 2 \mathbb{Z})$ and $H^{8}(F, \mathbb{Z} / 2 \mathbb{Z})$, of the $E_{8}$-type Lie algebras in the corner of Tits's magic square.

The algebraic group $\operatorname{Spin}_{14}$ has a special role in both quadratic form theory and exceptional algebraic groups. We developed some new techniques and classified all the mod 2 cohomological invariants of Spin $_{14}$-torsors (there are three nontrivial ones, in degrees 3, 6, and 7), continuing Jean-Pierre Serre's and Skip Garibaldi's tradition of classifying the cohomological invariants of various algebraic groups.

# Generalized polynomials and first steps towards multifractal analysis on Lie groups 

Arman Molla<br>University of Liège<br>October 6, 2022, 14:00 building B37, Allée de la découverte 12, 4000 Liège

## Thesis advisors: Prof. Dr. Samuel Nicolay (University of Liège)

## Summary:

The thesis explores two different subjects. The first one is the study of the Fréchet functional equation on Lie groups. In the classical setting, this equation entirely characterizes the polynomial functions. We hoped to generalize this way the notion of polynomials on Lie groups. We mainly focus on the local solutions of the problem. First, the regularity of the solutions is studied when the equation is studied on the space of locally essentially bounded function and even on space of distributions later. Then we describe explicitely the solutions in some precise classes of Lie groups (Abelian, nilpotent, solvable and semisimple cases). The second part of this thesis introduces the notion of pointwise Hölder regularity on unimodular Lie groups, a tool of main interest in multifractal analysis. We give a definition of these function spaces and a few useful characterizations using approximation by polynomials or by series of smooth functions having particular growth properties. At the same time, we also study the local version of these spaces and see how they are linked to the usual $C^{p}$ spaces from the manifold structure. The regularity of some simple functions is studied and in particular, a generalization of the continuous nowhere differentiable Weierstrass functions defined on compact Lie groups. We check on these examples that some coherent results with respect to the Euclidean case are obtained. Then, we also obtain various characterizations using wavelet-like transforms (the notion of wavelet depending of the sort of group) and we apply our results to see how our theory can be extended to the case of the unit sphere.

## 4 History, maths and art, fiction, jokes, quotations ...

### 4.1 Imaginary

See the poster at the end of this newsletter.

### 4.2 Adhemar's corner

Since last issue of this newsletter contained erroneously the same review as the previous edition, here follow two reviews of Adhemar. The first is on Terry Pratchett's Pyramids, a funny novel from the discworld series in which appears the greatest mathematician of all times. Secondly, Dana McKenzie gives an accessible account of recent progress made in mathematics and its applications in the 2019 collection of the series What's happening in the mathematical sciences.

## Pyramids by Terry Pratchett, Corgi, 1989 (341 p.)

Flatland by Edwin Abbott Abbott (1884) is well known, describing a two-dimensional world and how its inhabitats interact with the third dimension. It is often used as a model to explain how we, living in a three-dimensional world, should understand a fourth, or higher, spacial dimension. However, the idea of the author was to write a parody of the Victorian society. In a similar way, Terry Pratchett (1948-2015) invented Discworld, a flat planet in the form of a disc, and wrote in the period 1983-2015 not just one novella
 but 41 books about it which could be classified as "comic fantasy". His books were a hype for a whole generation and just like for Flatland, mathematician Ian Stewart, (and biologist Jack Cohen), collaborated with Pratchett in four books to discuss The science of Discworld.

There are obvious parallels between our society and the society of Discworld, only the events in the novels are following a Monty Python type of logic. The book Pyramids is the seventh book in the series, and I just single this one out because it does not happen often that you literally laugh out loud with its unbridled mathematical humour, and because it is featuring "the greatest mathematician of all times".

The story plays in Djelibeybi (meaning the baby of the river Djel) which is the Discworld analogue of ancient Egypt in our world. The main character is Pteppic (pronounce Teppic), son of the pharao who is raised as an assasin outside the country (in our world Assasins were an IS-like terrorist sect in Persia and Syria of the 11-13th century). When his father dies, he has to become the new king while his father is being mummified and a pyramid, the biggest ever, is built in his honour in 3 months time. Pteppic has some reasonable modern ideas, but the country is ruled by Dios, the high priest who "interprets" the ideas of the king in his own way to prevent any change at all. The previous king, assumed a semi-god, is supposed not to die and is still wandering around in the palace unable to communicate with the living, but having problems, his legs being stuffed with straw and his brains and bowels stored in a separate container. He and Pteppic try to prevent the construction of the pyramid which he never wanted, but Dios has it built anyway.

Ptraci was a concubine of the father and is supposed to commit suicide when the king dies. Because she refuses, she is brought to trial before Pteppic, who sets her free, but Dios interpreted this as that she will be thrown to the crocodiles the next day. Pteppic decides to rescue her during the night and flee with her to Ephebe (the analogue of ancient Greece). The energy concentrated in the pyramids makes them flare every 24 hours, but the enormous mass concentration of the new pyramid, which does not have a capstone yet, influences the space-time continuum and workers are getting déjà vu experiences and they are starting to remember things they were going to say in the future. Some cataclysm happens when space-time is rotated over 90 degrees, the gods and all previous rulers of the country return and the armies of two neighbouring states Ephebe and Tsort are starting a war (Trojan war) because Djelibeybi, which was a buffer state, has disappeared. Pteppic however returns, destroys the big pyramid, pushes Dios back to the first pharao, and everything returns to normal. He leaves the throne to Ptraci who introduces all the necessary reforms.

The adventure of Pteppic and Ptraci fleeing the country (part 2) and their stay in Ephebe (part 3) before their return are the parts of the book that made me want to review it here. They chose a camel as a vehicle for their escape. Camels are detached from their environment, their extensive digestive system gives them the time while ruminating to do mathematics and their curled lips allow them to solve quadratic equations. So they have a general aptitude to do higher mathematics. However they are smart enough to keep it a secret for humans since they do not want to end up being guinea pigs in their labs. And the escape animal they had chosen happened
to be the greatest mathematician of them all. Its name was You Bastard. Since it sensed the growing instability in the space-time continuum it starts calculating: "Let $V$ equal 3 . Let $\tau$ equal $\chi / 4$, cucuccud... Let $\kappa / y$ be an Evil-Smelling-Bugger differential tensor domain with 4 imaginary spin coefficients..." while Pteppic and Ptraci try to force it to move. It goes on computing while Dios is about to order soldiers to fire arrows that can "turn a charging hippo into three tons of mobile kebab", until "Let legs equal four..." and off they go. During the whole trip we learn about You Bastard's computations to control the bio-mechanics of its hazardous movements ". . let speed be gallop". Slowing down needs complicated instructions to each leg separately while the two passengers try to hold on its back. Camels have an advantage over humans to do mathematics because they do not have fingers. When humans are confronted with parametric differentials they have an instinctive tendency to start counting fingers. Camels don't. They just go with numbers and so they contemplate more easily "...Let $z=e^{i \theta}$ cudcudcud Then $d z=i e^{i \theta} d \theta$ or $d \theta=d z / i z \ldots$ ".

So You Bastard and its passengers escaped the collapse of the country which leaves only a crack in the surface as a fractal scar behind. They arrive safely in Ephebe, land of philosophers dressed in togas, bald headed and with long beards. The country is ruled by a Tyrant, but the Tyrant is elected every five years (not to be confused with what they do to tomcats, bulls, etc. to make them stop fighting and behaving more peacefully) using a system called mocracy (not to be confused with mockery). The first thing they experience is an arrow shot at them. You Bastard cudcudcud computes the point of impact in the sand before it actually happens. On a dune they see is a sign "axiom testing area" and a smaller one "caution-unresolved postulates". Behind it they meet Xeno (Zeno) whose slaves are shooting arrows at tortoises. Xeno claims these are the fastest animals ever. Shooting an arrow after it will never hit the target. But his companion Ibid (Aristotle) assures that tortoise is often on the menu and they are running out of tortoises. The one case where the tortoise has beaten the hare was because the hare was dead.

Pteppic is introduced to Xeno's colleagues during a Symposium, actually a knife-and-fork-tea. There is Copolymer (Homer), the greatest story teller, Iesope (Aesopt), the greatest teller of fables, Antiphon (Aristophanes), the greatest writer of comic plays, Pthagonal (Pythagoras), an acute man with an angle. The latter is grumbling over his pie because the ratio of the circumference over the diameter is not 3 or 3.5 or 3.3 for that matter, but no, it is 3.141 and lots of other figures. Pthagonal explains that pyramids are dangerous things. Their energy accumulates time and flares are needed to burn it. By building bigger and bigger pyramids,
 time has built up and it has turned space-time over 90 degrees. This rotation means that length is height, height is breadth, breadth is width and width is time. "There's four of these bastards". So you would get younger or older by walking sideways. Because that is obviously impossible, since "you would age another mile with every birthday", therefore, he concludes, the kingdom must have popped outside through a crack in space. "You can twist time and space all over the place if you are not careful." "Traveling in width is the real trick". And he concludes by asking: "Three point one four one, what sort of number d'you call that? Somewhere somebody should have built a universe with a decent, respectable value of pie, not some damn number that never comes to an end, what kind of -" and then he collapses because he had too much wine.

In a dream Pteppic meets Khuft, a camel herder, that became the first pharaoh after he discovered the river Djel that had not been there before (big bang). Something made it real. "I always thought it was the camels" says Khuft.

Adhemar Bultheel

What's happening in the mathematical sciences, vol. 11 by Dana Mackenzie, AMS, 2019 (vi+136 p.), isbn: 9781470441630.
I have reviewed in this newsletter volumes 7 (Sept. 2010) and 8 (March 2012) of this wonderful series What's happening in the mathematical sciences. I skipped volumes 9 and 10, but I think it is useful to bring this series once more to your attention. Dana Mackenzie brings again nine texts, richly illustrates, in which he reports about interesting mathematical achievements
 obtained in recent years, in a way that is accessible for a broad public. I restrict myself to inform you about some of them.

One topic is gerrymandering. This word is less familiar to us, but it is important for elections in the US where a system of 'the winner takes it all' is in vigor. It refers to US vice president Gerry (1812) and salamander. He signed a bill in which the boundaries of a voting district looked very odd like a salamander. Districts are redrawn after the updated sensus data. The idea was that by carefully designing the boundaries of the districts, this could bring an advantage in the election process. Suppose in a whole region the majority of votes goes to the Democrats, but subdividing the region into districts, one may do that in such a way that there are large districts where the republicans have a slight overweight, while the bulk of the democratic voters are packed in a few smaller districts, then the republicans can still win the most representatives in Congress for the whole state.


Michigan's 14 th congressional district 2020 A candidate winning with $50.01 \%$ is as good as one with $99.99 \%$ of the votes. Mathematicians were asked to produce a criterion by which this manipulative design can be condemned in court, but that is not so easy. One may think of minimizing the length of the perimeter, but then natural geographical restrictions are neglected and they are sensitive to small wiggles. Moreover, people do not live equidistributed. Then it has been tried to define redistricting such that it would not give a significant increase in efficiency of a vote for one party over the other. Whatever proposal that has been made, it remains a problem to find a fair criterion. The redistricting commission is often composed of the party that is in charge. Making that commission independent of the parties, would at least mend the cause of all this instead of fighting the symptoms.

Other contributions describe a PDE model for the dynamics of human metabolism, from which significant conclusion can be drawn concerning the effects of diets and weight loss. With the LIGO, VIRGO, LISA and other detectors, data are collected about gravitational waves. It turns out to be a gigantic numerical challenge to actually extract useful information about colliding black holes or neutron stars from the collected data which are, despite all precautions still noisy.

A cap set is a subset of the $n$-dimensional affine space $\mathbb{Z}_{3}^{n}$ over a field with three elements where no triple of points is collinear. The cap set problem is to find the largest possible cap set as a function of $n$. For $n=4$, this problem matches perfectly with the card game SET in which a card has 4 'attributes' (number, colour, shape, and fill) and each of these can take 3 values. So it has $81=3^{4}$ unique cards. The game is to recognize a threesome in which all attributes are different or


The 81 different SET cards the same (called a SET). A cap set is the largest possible number of cards that you can select that will not contain a SET. For $n=4$ this is 20 . Exact cap set sizes are known for small $n \in \mathbb{Z}_{\geq 0}(1$, $2,4,9,20,45,112, \ldots)$, upper and lower bounds for general $n$ are known, narrowing the gap is an ongoing research topic.

Another problem where mathematical modelling helps is how to distribute docking stations for citybikes. That is a standard mixed integer linear programming problem. If bikes accumulate at some stations, there is an extra cost of redistributing the bikes at the end of the day. But if it can be combined with an app that suggests the user to choose for a nearby station where the chance is higher to find a bike, this can improve a lot on the distribution cost and the number of bikes. Similarly an Uber taxi could possibly be shared by two customers to reduce the cost and the number of taxis. Perhaps taxis and bikes can be combined to improve the commuting problem of a city.

Expanders are sparse but strongly connected graphs. This is what one wants when modelling a brain: connect neurons fast to each other with few axions. The adjacency matrix of such graphs have a large spectral gap between the largest eigenvalue and the other ones. These graphs are also used to design of error correcting codes, recommendation systems, and knot theory.

One essay gives an introduction to Quantum Computing and operations on qubits. All possible logical functions of two bits can be built with only NOT and NOR gates. The problem now is to find such a similar set of two 'golden gates' for two qubits.

After Wiles proved Fermat's Last Theorem (FLT), mathematicians tried to solve generalizations like the Asymptotic version (AFLT) in more general number fields. Here it is explained how elliptic curves and modular forms are connected and how this contributes in reducing the search for solutions of the equation from an infinite set to a finite one. It all starts by solving a simple equation $x+y=1$ where $x$ and $y$ are rational numbers whose numerators and denominators are only divisible by prime numbers from a finite set.

Let me finish with another chapter on data analysis, which is another hot topic today. This is a remarkable story about the unexpected role played by topology in this field. Zeolites are porous materials that are used as adsorbents. The atoms of such material can be plotted as a well structured point cloud with empty holes. Assume now that we have an arbitrary set of data. Can we detect its topological structure? Can we find holes, and data that are closely connected? To detect such structures one has to draw balls around each point and consider points connected if their environments intersect. The radius of these balls can vary from very small, where there is no overlap anywhere, and as the radius increases, more and more points and components become connected, holes are formed, but as the radius increases more, everything will be connected, but the big 'holes' will be most persistent. This needs some

zeolite

data introduction to persistent homology. The idea is to see which topological invariants are most persistent under increasing distortion of the space. Invariants are for example the Betti numbers giving the numbers of connected components $\left(\beta_{0}\right)$, the number of tunnels $\left(\beta_{1}\right)$, and the number of cavities $\left(\beta_{2}\right)$. There are other ways of topological data analysis, using successive filters, but the result is similar. One arrives mathematically at a 'simplicial complex'.

This kind of analysis knows many applications. We can think of searching for zeolites with extreme porosity. It was used on clinical data to identify defects in a variety of genes that can be linked with the different types of diabetes, which defined more appropriate treatments. It was detected which brain cells, not in the visual cortex, were active when rats are maze running. Given a two-dimensional structure of a molecule, it is a challenge to find out how it will fold and what its physical properties are. Classical methods of computational chemistry keeps track of the thousands of atoms, but the data reduction via the topological analysis leads to a much simpler solution.

Adhemar Bultheel

## ALTAÏR

Centre d'Histoire des Sciences et des Techniques reconnu par le Conseil de la Recherche de l'ULB

## PROGRAMME DES CONFERENCES DE L'ANNEE ACADEMIQUE 2022-2023 :

8 octobre 2022 : Anne De Wit (ULB) La séquestration géologique du CO2

26 novembre 2022 : Jean-Pierre Devroey (ULB) L'apport des données climatiques à l'analyse historique (et vice-et-versa). Une étude de cas à l'âge de Charlemagne (740-820)

10 décembre 2022 : Jean-Jaques Heirwegh (ULB) Balthazar de Monconys (1611-1665), voyageur et «curieux de la nature »

11 février 2023 : Sophie Van Eck (ULB) Constellations de satellites

11 mars 2023 : Samuel Fiorini (ULB) Preuves, algorithmes et complexité : en safari autour de $P$ et NP

18 mars 2023 : Alain Jorissen (ULB) Le satellite Gaia, un coffre aux trésors galactiques

22 avril 2023 : Michel Godefroid (ULB) L'Atome: un outil merveilleux pour sonder et comprendre notre Univers

Horaire : le samedi matin de 10 h à 11 h 30
Lieu : ULB Campus Plaine, Boulevard du Triomphe, Forum, auditoire E - un changement de local éventuel sera communiqué aux auditeurs inscrits Inscription requise sur https://forms.gle/BgAt33USH1vRmTfaA

Tout public : $5 €$ par séance Étudiants : entrée gratuite Membres d'Altaïr : entrée gratuite (cotisation annuelle 12,50 € à l'entrée ou à verser sur le compte d'Altaïr BE45 00123409 1489)

Renseignements : Alain Jorissen (Alain.Jorissen@ulb.be) ou 026502834 et Luc Lemaire (Luc.Lemaire@ulb.be) ou sur les sites : http://altair.ulb.be et. https://www.facebook.com/Altair4ULB/

