



Newsletter

BELGIAN MATHEMATICAL
SOCIETY

159, September 15, 2026

Comité National de Mathématique

CNM

C W M
N

NCW

Nationaal Comité voor Wiskunde

**Newsletter of the Belgian Mathematical Society
and the National Committee for Mathematics**

Belgian Mathematical Society ASBL/VZW
ULB Campus Plaine, C.P. 218/01,
Bld du Triomphe, B-1050 Brussels, Belgium

Website: bms.ulb.ac.be

Newsletter: andreas.debrouwere@vub.be



By Andreas Weiermann

Contents

1	News from the BMS & NCM	2
2	Meetings, conferences, workshops, lectures, ...	3
3	PhD theses	6
4	News from the universities and other societies	7
5	Book reviews	8

The next edition of this newsletter will appear on 15 November. Please send your content to andreas.debrouwere@vub.be.

Foreword by the president of the BMS

Dear BMS members,

Before we come to the BMS events planned for the rest of this year, let us recall our main event in the spring: The joint BeNeLux Mathematical Congress on April 7-8, 2026, in Antwerp was a full success with more than 220 present participants on each day who enjoyed the talks of the 4 plenary speakers (Jessica Fintzen, Kathlén Kohn, Christophe Thiele, Nathalie Wahl) and took part in the 25 parallel sessions and the poster session (with best poster awards) on a broad variety of topics, for details see the [website](#).

In the second half of 2026, our main event will take place on December 16, 2026, when the BMS will organize a meeting on the interplay between mathematics and AI. More information about the event will be announced soon on our [website](#) and facebook page.

Moreover, the call for the BMS Young Scholar Award 2026 is upcoming, a yearly prize awarded by the BMS board to junior mathematicians. More information and a list of previous winners can be found on our [webpage](#).

Best wishes & hope to see you all on December 16,
Sonja Hohloch

1 News from the BMS & NCM

1.1 Upcoming Events

On December 16, 2026, the BMS will organize an event on mathematics and AI. More information about the event will be announced soon on our [website](#) and facebook page.

The BMS will soon launch a call for the Young Scholar Award 2026. This is a yearly prize awarded by the BMS board to junior mathematicians, please visit our [website](#) for more information and a list of previous winners.

1.2 Bulletin of the Belgian Mathematical Society - Simon Stevin

As a member of the BMS you have electronic access to all electronically available issues of the Bulletin of the Belgian Mathematical Society - Simon Stevin, free of charge. If you have any trouble logging in or accessing the journal, please contact customer_support@projecteuclid.org.

Content Volume 33 (3) July 2026

- F. Delbaen, *Martingales with independent increments*, [Article](#).
- G. Schindl, *Generalized upper and lower Legendre conjugates for weight functions*, [Article](#).
- C. Hughes, A. Miranda, *Colimits of internal categories*, [Article](#).
- M. La Barbiera, R. Moghimipor, *Algebraic and combinatorial invariants of monomial ideals in two sets of variables*, [Article](#).
- Y. E. Felke, T. Mengestie, M. H. Takele, *Closed range and spectral of superposition operator on harmonic Fock spaces*, [Article](#).

For the table of contents of previous issues, see <https://projecteuclid.org/all/euclid.bbms>.

2 Meetings, conferences, workshops, lectures, ...

Miniworkshop “Geometry & Mechanics: Differential Systems and Related Topics”

University of Antwerp, September 17-18, 2026

This 2-day miniworkshop on September 17-18, 2026, in Antwerp, Belgium, aims at bringing together junior and senior researchers interested in various aspects and applications of geometric mechanics, differential systems, and related topics. It ends with the public PhD defense of Kenzo Yasaka.

The list of speakers includes Malika Belrhazi (Antwerp, Belgium), Eduardo García-Toraño Andrés (Universidad Complutense Madrid, Spain), David Martín de Diego (ICMAT, Madrid, Spain), Geoff Prince (La Trobe University, Melbourne, Australia), Kenzo Yasaka (Antwerp, Belgium), and Federico Zadra (Antwerp, Belgium).

More information can be found on the workshop webpage: <https://www.uantwerpen.be/nl/personeel/sonja-hohloch/private-webpage/conference-workshop/diffsystems-2026/>

ATLAS: Algebraic thoughts looking at space

Université Libre de Bruxelles (ULB), Brussels, September 29, 2026

A symposium in honor of Michel Van den Bergh on the occasion of his official retirement.

Speakers: Pieter Belmans, Tom Bridgeland, Bernhard Keller, Anya Nordskova, Maxim Kontsevich.

Organisers: Wendy Lowen, Špela Špenko.

More information can be found at <https://atlas-26.ncag.info/>.

Meeting of the FNRS Contact Group in probability

Brussels, October 15–16, 2026

We are delighted to invite you to the next meeting of the FNRS Contact Group in probability, which will take place on 15–16 October 2026 in Brussels.

The meeting will be held at Château Tournay-Solvay. Information will be posted on the meeting webpage: <https://sites.google.com/view/fnrs-cg-probability/2d-meeting-2026>.

Please register using the following form: <https://forms.gle/2Ksud5L1D6Gvzpx96>.

Feel free to circulate this announcement widely, including among your students. We would be delighted to welcome a large audience.

We hope you will be able to join us.

All the best,
Céline Esser and Yvik Swan

Advanced Studies in Harmonic Analysis and PDE: Research Perspectives from East Asia

Ghent University, October 27–30, 2026

Advanced Studies in Harmonic Analysis and PDE: Research Perspectives from East Asia is a 4-day specialized doctoral course bringing together leading experts from East Asian and Flemish universities. The programme consists of structured mini-courses, a panel discussion on current research directions, and a poster session offering opportunities for feedback and networking. The course aims to provide advanced training in harmonic analysis and PDE while promoting international scientific exchange and collaboration.

Participants will gain advanced insight into selected current topics in harmonic analysis and partial differential equations, including contemporary methods and open research directions. The course will also provide broader perspectives on research trends across Europe and East Asia, while highlighting opportunities for future collaboration. The poster session offers participating PhD students an opportunity to present their research, discuss mathematical ideas, and receive constructive feedback from peers and senior experts from East Asia and Flanders.

Everyone is welcome to attend, and PhD students in particular are warmly encouraged to participate. Participation is free of charge, thanks to support from the Doctoral School and the Flemish Government.

Registration link: <https://forms.gle/UYonBQ2AfdWyEWe38>

Website: <https://analysis-pde.org/advanced-studies-in-harmonic-analysis-and-pde-research-perspectives-from-east-asia/>

Mutation en Mathfest — Wiskunde, Kunst & Technologie

Gent, 20-22 november, 2026

Mutation keert terug van **20 t/m 22 november 2026** in het Wintercircus in Gent! Mutation is een participatief festival op het kruispunt van kunst, technologie en samenleving. Organisaties, makers en denkers bouwen samen bottom-up aan een programma vol workshops, expo's en performances.

Lees er alles over op www.mutationfest.be of <https://mathfest.be>.

Hoofdthema 2026: Why Tech? Dit jaar onderzoeken we technologie met nieuwsgierigheid en stellen we de vraag hoe innovatie onze maatschappij, identiteit en creativiteit beïnvloedt. Binnen de drie themalijnen springt er voor wiskundeliefhebbers één in het bijzonder uit: **(Algo)rhythm**. Binnen deze lijn verkennen we de ontmoeting tussen menselijke intuïtie en wiskundige systemen. Hoe beïnvloeden wiskundige patronen, algoritmes en data de manier waarop we bewegen, denken en kunst maken?

MathFest op zondag 22 november. Noteer alvast **zondag 22 november** in je agenda: deze dag staat als familiedag in het teken van **MathFest!** Een uitgelezen kans om te ontdekken hoe wiskundige logica en creatieve technologie elkaar versterken.

Muziekoproep voor MathFest. Ken jij een lied met een wiskundige link, in de tekst, het ritme, de structuur of ergens anders? Scan de QR-code en stuur je nummer in, samen met een korte uitleg. Tijdens MathFest brengen we alle inzendingen samen in een playlist.

Wil je nog een stap verder gaan? MathFest organiseert een pitchbattle met als centrale vraag: **welk muzieknnummer heeft de mooiste wiskundige link?** Wil je de uitdaging aangaan, geef dit dan aan wanneer je jouw liedje instuurt. We selecteren acht deelnemers, die elk drie minuten krijgen om het publiek te overtuigen. Zij strijden om de eer én om prachtige prijzen! Wil je deelnemen aan de pitchbattle? Dien je inzending dan uiterlijk **zondag 1 november** in.



Doe mee als partner of maker. Wil je zelf een activiteit, workshop of inzicht bijdragen aan het programma? De Open Call staat open tot **20/09/2026**.

Meer info?

Surf naar www.mutationfest.be/en of mail info@mutationfest.be.

VUB Algebra Seminar

<https://leandrovendramin.org/team/seminar.html>

Analysis & Geometry Seminar (UAntwerpen)

This is the weekly research seminar of the analysis & geometry-interested people in the Antwerp. During the semester, we will have once per week a research talk in analysis and/or geometry and/or related topics. The list of speakers comprises researchers from Antwerpen as well as other universities. More details can be found on the seminar webpage: <https://www.uantwerpen.be/nl/personeel/sonja-hohloch/private-webpage/seminars/analysis-geometry/>.

3 PhD theses

On the symplectic geometry and topology of (hyper)semitoric systems

Pedro Santos
University of Antwerp
June 20, 2026

Thesis advisor: Prof. Dr. Sonja Hohloch

Summary: This PhD thesis studies the symplectic topology of the fibers of semitoric systems, and introduces new symplectic invariants for simple hypersemitoric systems and simple hyperbolic-regular lines in Hamiltonian S^1 -spaces: First, we generalize McDuff's method of probes, and use this to study the (non)displaceability properties of the fibers of semitoric systems. Furthermore, we prove that any nonsimple focus-focus fiber of a semitoric system is nondisplaceable. Then, we introduce the affine invariant for simple hypersemitoric systems, generalizing the semitoric polytope invariant of semitoric systems. Furthermore, we introduce a J -height invariant and multiple J -Taylor series invariants for simple hyperbolic-regular lines in Hamiltonian S^1 -spaces.

Exterior differential systems on Lie algebroids and their applications

Kenzo Yasaka
University of Antwerp
September 18, 2026

Thesis advisors: Prof. Dr. Sonja Hohloch, Prof. Dr. Tom Mestdag

Summary: In recent years, there has been increased interest in a coordinate independent geometric theory for studying solutions of systems of PDEs, known as the theory of exterior differential systems (EDS). This theory originates from the ideas of several mathematicians, such as Pfaff, Darboux, Goursat and, most prominently, Cartan. Here, the PDEs are represented by a collection of differential forms on a manifold, satisfying the condition that the collection is closed under exterior differentiation. The main motivation to study EDS is to compute integral manifolds, which are submanifolds satisfying the property that all differential forms in the collection vanish. Since any system of partial differential equations can be written as an EDS (with an independence condition), finding solutions to a system of PDEs can be given the desired geometric interpretation, namely, finding integral manifolds of its associated EDS.

In this thesis, we extend the theory of EDS from manifolds (and their tangent bundles), to Lie algebroids. To be more precise, we extend the theory so that instead of limiting ourselves to finding submanifolds such that a collection of differential forms on a manifold vanishes (i.e., a form on the tangent bundle), we will find submanifolds so that certain forms on a vector bundle vanish. However, to make sense of the fact that the collection of forms should be differentially closed, we need to assume that this vector bundle has some additional structure, namely, that of a so-called Lie algebroid, where an exterior derivative operator is defined.

First, we focus on existence results for integral manifolds, in the case where the Lie algebroid is transitive. In particular, we generalize two versions of the celebrated Cartan–Kähler theorem from EDS on manifolds to this setting. We then motivate our extension by giving many applications. We find applications in b-symplectic geometry (using an EDS on a b-tangent bundle), and in group-invariant solutions to dynamical systems with symmetry (using an EDS on an Atiyah Lie algebroid). We conclude with a detailed discussion of the invariant inverse problem, where we demonstrate that it can be reformulated as an EDS problem on a very specific Lie algebroid, called the IP Lie algebroid.

Harmonic Analysis on Euclidean Domains with Global Structures

Arne Hendrickx
Ghent University
October 26, 2026

Thesis advisors: Prof. Dr. Michael Ruzhansky and Dr. Zhirayr Avetisyan

Summary: This thesis develops harmonic analysis on Euclidean domains endowed with suitable global structures. The underlying principle is that the geometry and global structure of a space can be incorporated into the Fourier-analytic and pseudo-differential tools used to study functions and operators on it. While this principle is well established in harmonic analysis on groups, where the algebraic structure of the group encodes the relevant symmetries, we investigate a complementary perspective in which the underlying space remains a Euclidean domain. Two particular types of global structure are considered, namely periodicity induced by lattices and global nonlinear coordinate systems relating a domain to $\mathbb{R}^d$. The resulting framework is applied to the study of boundedness of Fourier multipliers, pseudo-differential operator theory, and partial differential equations, including the existence of fundamental solutions and boundary value problems on a broad class of Euclidean domains.

4 News from the universities and other societies

Royal Society special issue - Derived Lie algebras in geometry and topology

Royal Society Publishing has recently published an **OPEN ACCESS** theme issue of *Philosophical Transactions A* under the ‘Subscribe to Open’ 2026 initiative: **Derived Lie algebras in geometry and topology** compiled and edited by D Lukas B Brantner and Gijs SKS Heuts, and the articles are FREE to access at www.bit.ly/TransA2328.

A print version is also available at the special price of £40.00 per issue from sales@royalsociety.org.

5 Book reviews

Below are reviews by Adhemar Bultheel of the following books:

- Jim Al-Khalili, *On time. The physics that makes the universe tick*, Princeton University Press, 2026, 360 p., isbn: 978 069126703 6 (hbk).
- Marcus du Sautoy, *Blueprints. How mathematics shapes creativity*, Fourth Estate, 2025, 384 p., isbn: 978 000868500 3 (pbk)

Enjoy reading them!

Jim Al-Khalili, **On time**. The physics that makes the universe tick, Princeton University Press, 2026, 360 p., isbn: 978 069126703 6 (hbk)

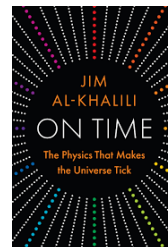
Al-Khalili is a physicist and science communicator. He features regularly on BBC radio and TV and he has published several books on paradoxes, quantum theory, physics, and history. I liked this book in particular because he explored deeply the enigmatic nature of time: mathematics does not seem to explain the arrow of time because formulas are time-symmetric. Einstein introduced it as a fourth dimension, but it definitely is different from the three space dimensions. And our personal experience is that it runs faster when we have fun, and slower when we are waiting. Older people have the feeling that times runs faster than when they were young. So is time something people generate in their brain, or does time exist independently of our perception? Is it infinite and has it a beginning and an end? All questions considered in psychology and philosophy, but where we hope that science can give an answer, So far we are still looking for accurate models of the human brain and we are missing the unification of quantum theory and Einstein's (general) relativity theory that should become a unified theory of everything (ToE). Where do we come from and what future is our cosmos running into? In other words, time touches on all the big questions. And Al Khalili investigates all their possible answers and gives his own opinion on which answers could be the right ones and which ones to exclude.

Reading this book is a mind blowing experience that makes it painfully clear that there is so much that we still do not know, and although there is not so much explicit mathematics, the mathematicians are figuring in the background from Greek philosophers over Laplace and Newton, to Einstein, Hawking, and Penrose. Eventually quantum gravity and the ToE that should hold both for the extremely small and the extremely large will obly be solved when the appropriate mathematical framework will be developed.

The first chapter is an exploration of the problem or rather the multitude of problems. While it is clear that in 3D space all the positions do exist simultaneously and we can move from one position to another. If we leave a room, we know it is still there and we can return if we want to. But that is not possible for time. We can only undergo time. So, do all time instances exist when we are not living it? The past is gone, the future not here yet. Does time exist physically, or is it something that emerges from our experiences, like temperature is emerging from the activity of atoms in a material? We experience time as either the past which we know (remember) partially, and we can expect what's going to happen in the future, but we do not know for certain. It does not exist yet or does it? And what is "now"? Is it a cut between the past and the future? We all have an intuitive idea what is "now", how long it remains "now"? Philosophers distinguish between *eternalists* who claim that time is real and all time instances exist always, or *presentists* who claim that only the present exists and that time (and its consequences like change, causality, etc.) is emergent. Think of Zeno's paradoxes of motion. And there is of course the arrow of time: does time flow by while we are static, or is time static like space and are we moving through time like we move through space at a rate that we cannot control neither in direction nor in speed. Different arguments have been given to explain the arrow of time, but none is absolute, And finally, is time continuous or is a sequence of time-quanta?

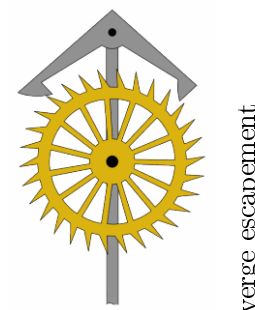
I can give you Khalili's answers to some of these questions. This is not a spoiler because he also gives them at the beginning of the book: The time flowing is an illusion; time is real, i.e., not just an emerging experience; and the arrow of time has many appearances, but there is a "master arrow" given by nature which is generated by quantum decoherence which relates to the *past hypothesis* (i.e. time started at a moment of extremely low entropy, which is the Big Bang ($13,8 \times 10^9$ years ago), and entropy has ever increased since).

The full explanation and his arguments for these answers he gives in the subsequent chapters



Jim Al-Khalili

subdivided into four parts and a bonus chapter in which he discusses different aspects of time. The first part is the time t as it appears in formulas, i.e., as a parameter or a label. This chapter has a strong historical component describing instruments to measure time, i.e., methods to capture change, from solar time-keeping over sand or water clocks to clocks with a pendulum and a verge escapement; introduction of the GMT, quartz crystals, atomic clocks and the most recent optical lattice clocks with a resolution of 1 attosecond (10^{-18} sec) with a possible drift of 1 sec in 30×10^9 years. And yet, this is still far away from the Planck time ($15,39 \times 10^{-44}$ sec), the smallest meaningful time interval. It relates to the Planck length which is the distance traveled by light in 1 sec (1.61×10^{-35} m). Beyond these limits, time and space make no sense any more. Chapter 3 is a discussion of how we experience time to be slow or fast. Two possible explanations are the fraction of life: later in life a time interval is a smaller portion of your past and therefore experienced as being shorter, i.e., time runs faster. Another explanation is the ‘new experience theory’ which states that if our brain has to process more new experiences then time seems to pass slower, while it seems to pass faster when experiences are familiar. That explains why driving to a place you never went before seems to take longer than the way back. However it does not explain while sitting in the doctor’s waiting room takes so long.



Chapter 4 is about if and how we can predict the future. Laplace viewed the world as a mechanical machine. Everything is predictable as long as we know the state of all the variables, the future can be computed by solving an initial value problem. But the current state can also be used as the state reached from the past. The math works in both time directions, which seems to contradict the arrow of time. Of course there is the chaotic behaviour as a consequence of the butterfly effect and since one electron at the boundary of the universe can influence (a system is never completely isolated) we should in fact know the state of everything in the whole universe if we want to predict accurately. That would rule out the existence of free will. But there are many other limitations like the Heisenberg uncertainty principle and whether or not information can be lost when it is absorbed by a black hole. The latter made Hawking and Penrose believe at some moment that the cosmos was not deterministic.

In the second part of the book, time is considered as a dimension. This is the place to explain the very strange time effects that follow from accepting that light is traveling at a constant speed in vacuum: the special and later the general relativity theory of Einstein is explained and all the time effects that these imply. Space and time become one 4D universe of spacetime, which leads to several paradoxes like the pole in the barn or the twin paradox if one is approaching the speed of light in special relativity or if one moves near a black hole in general relativity. Here is mathematics with tensors and manifolds an essential tool but Khalili explains the effects elegantly avoiding the formulas. But still there is a discussion about the cosmic microwave background, that proves (if we accept the cosmological principle, i.e. that the universe is homogeneous and isotropic) the existence of a Big Bang, the expanding of a cosmos where space is infinite without boundary. Also the Friedmann-Lemaître equations derived from Einstein’s field equations are discussed. These split spacetime again in space and time ($4 = 3 + 1$) in which space itself is flowing in time. There is a cosmological time which is the expansion of space = change = time.

Part three is about time as an arrow. This cannot be discussed without a definition and discussion of entropy, the beginning and the end of time, i.e. about the origin and ending of the cosmos. The entropy discussion is kept at a level of statistical mechanics. Khalili sees no problem in the fact that the symmetry and chaotic movement at low level will allow a directional effect at a larger scale. If a black hole increases its entropy, will it ever reach a maximum and fall back into an equilibrium, i.e., low entropy state? Such possibilities are also open for the whole cosmos: it may end in a Big Crunch (i.e. the expansion reaches a maximum, time is reversed and it shrinks back into a Big Bang situation), or will it end in a Big Freeze (it keeps

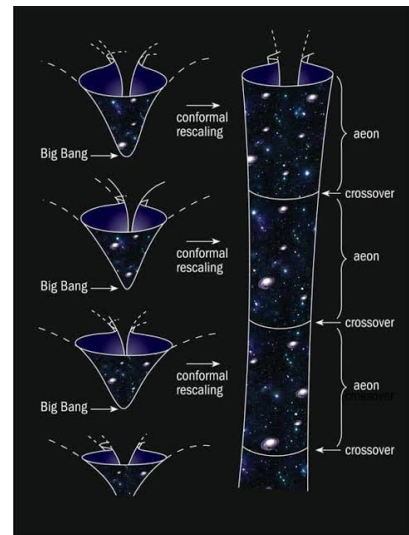
expanding and will end in a dark and cold void), or Big Rip (the cosmos keeps expanding at increasing speed till it is torn apart) or a Big Bounce (it oscillates and shrinks after reaching a maximal expansion after which it starts expanding again). A model of the latter type was proposed by Thomas Gold (1960) in which time is reversed after a while, or there is Penrose's conformal cyclic cosmos where after a massive black hole has swallowed all mass, the Hawking leaking will recreate a situation of a new Big Bang at a new cosmic scale which can be repeated indefinitely. Why has there been a Big Bang and a beginning of time? Why were, at some point, all the physical constants just right to generate the universe that we live in? Thomas Hertog (Leuven) proposes that maybe all possibilities were present in quantum superposition, but because we are part of the system, our observations broke the superposition and generated the constants we know. We created our own universe as it is.

In a fourth part the reality of time is discussed. Here Khalili gets his final answers that he derives from quantum theory and quantum gravity for which some promising theories of Loop Quantum Gravity (LQG) and Causal Set Theory (CST) are explored. In CST, it is assumed that spacetime is fundamentally discrete, implying that no singularities actually exist and that causality and A happening before B means that B is in the future light cone of A and this generates all physical measurements. The LQG theory also proposes a granularity of spacetime. It is related to the Wheeler-DeWitt equation which is a relativistic kind of Schrödinger equation. However at this point it is still unclear if any of these theories is to be accepted. As mentioned above, the conclusion is that Khalili accepts time as being real and that the arrow of time comes from the irreversibility of quantum decoherence: once an observation is done, the quantum superposition disappears and cannot be undone.

I have mentioned many things in this review, but many more are discussed by Khalili (Maxwell's demon, Boltzmann brain, Olbers and Loschmidt's paradoxes, etc.), and if all this sounds very complicated, if you read this book, it sounds all so simple and logical until you are surprised by some paradox that undermines what you believed to be correct so far. You are guided and showed around in a world that is very removed from your daily life and yet it is a world we are all part of.

I appreciated how Khalili really investigates all the possibilities and all the solutions that have been proposed, accepting or disposing them using reasonable arguments, never being dull or using high brow complicated expositions. He is really entertaining the reader.

As a bonus, Khalili provides a desert with a chapter about time travel. Will it ever be possible? What kind of paradoxes are awaiting us if we travel in time? Perhaps, it are not the laws of physics but the laws of logic that prevent time travel. The past can define the future but the future cannot influence the past. If there are two versions of yourself, this would add mass to the universe, and if you iterate we could create an impossible infinite mass. You cannot remember meeting your future self (bilking paradox) etc. It requires a multiverse to travel via wormholes in a block universe (where past and future coexist like space). Kurt Gödel wrote a paper in which he describes a time loop in a rotating universe. He obtained it by solving Einstein's field equations, so it was mathematically correct but physically impossible. It was not written because he believed it, but he wanted to show Einstein (they were both in Princeton) that objective time did not really exist. Similarly other models have been computed compatible with Einstein's general relativity, but they were all very unrealistic. The most pressing question is: if ever time travel will be possible, then where are all these time travelers? They do exist, but so far only in literature and films and in our imagination.



conformal cyclic cosmos (R. Penrose)

Adhemar Bultheel

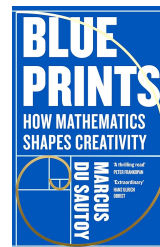
Marcus du Sautoy, **Blueprints**. How mathematics shapes creativity, Fourth Estate, 2025, 384 p., isbn: 978 000868500 3 (pbk)

The book starts by explaining the title. A blueprint was/is a cheap copy of a plan that was made by for example architects. It had white lines on a blue background because of the copying technique. It gives a general outline of the plan showing the basics of the construction. This is the topic of the book: sketching the basics of what supports the creativity in mathematics and in a piece of art, and there are many similarities and parallels to be pointed at. A construction with only two fulcrums is unstable, so the third one that supports the exposition is nature. Clearly artists are inspired by nature and mathematicians are a basic instrument to understand and explain nature, but in this book, nature is somewhat in the background. Whether mathematics is invented or intrinsic in nature to be discovered, is left open.

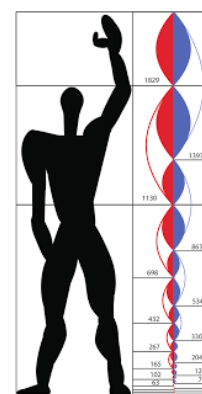
Du Sautoy has selected nine mathematical blueprints to make his point. Not surprisingly these topics involve numbers and geometry, which are close to the author's own work as a mathematician. The first is **prime numbers** which starts with an analysis of *Quartet for the end of time* by the composer Olivier Messiaen in which many prime number were used. But primes are also useful to optimize the sound quality in opera or music theaters and Shakespeare used 7 or 11 syllables in his verses to stress the important ones like the 11 in *To be or not to be that is the question*. But many more musicians and writers are presented by du Sautoy to illustrate how they used primes in their work, Nature is represented by the well known cicadas that come to life in prime number cycles to minimise the chance to be confronted with predators, or the number of flower petals, or spirals on pine apples, sunflowers, etc. Primes are considered so fundamental that they were used in messages to make possible contact with extraterrestrial aliens.

The second blueprint is **the circle**. It has been part of civilisation since prehistory as testified by constructions like Stonehenge, and the circle is the cradle of π , the most iconic number of mathematics. The opening is a discussion of the work of Jorge Luis Borges, among which his influential short story *The circular ruins*. The idea of circularity is also related to infinity, which is a recurrent theme in his stories. The 3D generalization has been considered since antiquity as symbol for perfection. Architects struggled with its construction as a semiball as a dome because it is not self supporting. It requires the inverted catenary as proved by Robert Hooke in 1675 to construct a stable arch or a dome, which was for example used by Brunelleschi to construct the dome of the cathedral in Florence and by Gaudí in the *Sagrada Familia* cathedral. In fact the Duomo in Florence has 3 layers: the outside one is spherical, supported by a conic one that is carried by the inner self supporting one. Also Étienne-Louis Boullée was a French architect from the Enlightenment who wanted to construct a spherical monument to honor Newton (cfr, Peter Greenaways *The belly of an architect*). Bach was a master of using circular loops in his compositions, but there are also films and stage plays that are using circular ideas or ideas related to a Möbius strip or a torus. For example the play *I is a strange loop* written and performed by du Sautoy and Victoria Gould (still to be seen on YouTube).

Blueprint number three is about **Fibonacci numbers**. These, as well as the limit of their ratio, the golden section, are well known to appear often in nature. Here the architect Le Corbusier who constructed *L'Unité d'Habitation* in Marseilles, a social housing project, all according to the ratios defined by the Fibonacci numbers which he subdivided into a red sequence (starting with 6 and 9) and a blue sequence (starting with 11 and 18). He also represented these with his *Modulor*, a sort of Vitruvius kind of subdivision of the human body. Du Sautoy even includes a proof that $\sqrt{2}$ is irrational. It is also explained why these Fibonacci numbers important to define music's chromatic scale and how they pop up in describing the pos-



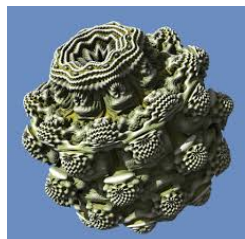
Marcus du Sautoy



Modulor

sible rhythms of verses of a fixed length in Indian scriptures where syllables can be short or long (= 2 short ones) as well as in Phillip Glass's opera *Einstein on the beach* who got it from Indian music.

The golden section is blueprint number four, which opens with a thorough discussion of Eleanor Catton's novel *The luminaries*. But it is also a favorite number of nature in biology, it is present in the design of our paper format A5, A4, A3,... in the music of Debussy and Bartók, and in Dan Brown's *Da Vinci Code*. Much originated from the work of Pacioli and the *divina proportione* of Leonardo Da Vinci himself that have influenced many artists of subsequent centuries. Architects considered the ratio as optimal and it became later an important number just like number 3 for freemasons, and hence also in Mozart's *Die Zauberflöte* and texts by Goethe.



Mandelbulb

Fractals are chosen as blueprint five. This requires some definition of how one can obtain the fractal dimension of a curve like Koch's snowflake by covering it with squares and how to extend it to shapes and characteristics of a painting like the unique fractal dimension that can reveal that a painting of Jackson Pollock 'the dripper' is an original. Fractals are also omnipresent in nature and since Benoit Mandelbrot discovered the fractal dimension, the set named after him with its fascinating self-similarity became omnipresent as colourful projections in music videos in the 1980's and later became an essential tool to generate natural looking landscapes in Pixar movies. Some fractal features can be found in ceiling decorations in the Alhambra in Granada or in the gardens of the Taj Mahal or Japanese gardens and it has been applied in music as well like in the *Goldberg variations* and in a stage play such as Tom Stoppard's *Arcadia* that has a mathematical content and form.

Blueprint six are the Platonic solids. Somewhat surprisingly, this starts with the choreographic notation of Rudolf von Laban who organized a spectacle before the opening of the Berlin Olympic Games in 1936. The structures are also the constellations of crystallography, the shapes of viruses and nanoparticles. Plato used them to represent 4 basic elements while the fifth stands for the cosmos, and they are found as neolithic trixels in Scotland. They were re-discovered during the Renaissance because of their symmetry and they appear in drawings of Leonardo Da Vinci or paintings using the new tricks of perspective to show the craftsmanship of the artist. Their rich symmetry was idealized by Kepler and Dürer to explain nature and the planetary system. Also Dalí used a dodecahedron in his painting *The Last Supper*, and the tesseract to represent a hypercube in *Crucifixion*. Dalí's last paintings were related to chaos theory which he learned from discussions with René Thom, *The tesseract* is also the title of a novel by Thomas Garland where dimensions were replaced by narratives.

The symmetry as blueprint seven has as main story the work of Raymond Queneau and of the French group Oulipo that he started. The idea is to create texts subject to certain constraints such as exploring all possible permutations in rhyme schemes, which resulted in *Cent mille milliards de poèmes* by cutting up the lines and allowing to read them in any order or *La disparition*, a book without the letter 'e'. The Greek composer Iannis Xenakis wrote *Nomo Alpha* for cello which is based on all possible symmetries of a cube and the decorations in the Alhambra is a huge collection on symmetries.

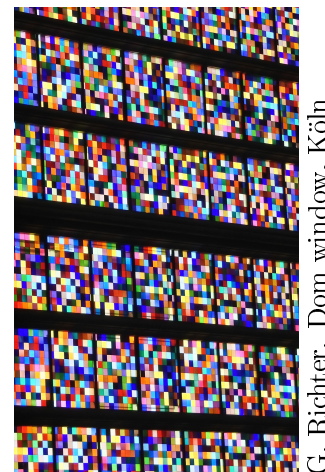


Heydar Aliyev Center, Baku

With blueprint eight about hyperbolic geometry, we are back into architecture with Zaha Hadid (Aquatic Center, London, Olympics 2012, Heydar Aliyev Center, Baku, Azerbaijan or Port Huis in Antwerp) and the curved mirrors of Anish Kapoor abandoning all straight lines while M.C. Escher explored hyperbolic geometry in its representation using the Poincaré disk while the Wertheim

sisters crocheted the hyperbolic geometry in 3D. Einstein's geometry of cosmic space-time is not Euclidean. Du Sautoy explains a Seifert-Weber space which is obtained by connecting the opposite sides of a dodecahedron which is only possible after a rotation. Emily Howard and du Sautoy show how mathematical ideas can be translated into music in a string quartet *Four musical proofs and a conjecture* (check YouTube!)

The final blueprint is randomness, which is a bit different. Like the lines of Queneau's poems, novels, and stage plays were written such that they can be read or performed in any order that could be decided by the reader or the public, or just by rolling a die. Chance can be used during the creation, or during the consumption, or the artist can design the algorithm such that each performance should be different. John Cage's *Music of changes* is an example who tossed a coin during the composition. Randomness doesn't look always very random. E.g. in a random colour sequence a sequence of equal colours is perfectly possible to appear, but does not look very random, will then the artist make some modifications? Gerhard Richter created *4900 Farben* consisting of 196 square panels with 5 by 5 squares of random colours, with the instruction that they can be hung up during the exposition in a random set of square arrangements of these panels.



G. Richter. Dom window, Köln

He also created a stain glass window for the Cologne cathedral with random colours for which he had to make some rearrangements to make it look more random. Pierre Boulez generated music that is generally controlled but depends locally on chance, a technique he called *Aleatorik*. In my opinion this is very similar to mobile or kinetic art by Alexander Calder and others. And isn't this the way how nowadays GenAI is producing content following our instructions.

In a concluding 'Finale' du Sautoy reflects on what other blueprints could have been selected and in how far the blueprints are really realised in the different settings. Mathematics is idealizing nature, since there is no perfect circle of perfect fractal in nature and is the randomness of quantum theory really perfectly random? In any case mathematics is based on our attempt to explain nature, but how much of mathematical rules will nature follow. So we are back at the question if mathematics is discovered or invented?

Nature is free in its evolution in the sense that at some time in the past some species developed and others have disappeared. Scientists in general have to conform with nature while mathematics writes its own rules and can develop a theory that does not necessarily represent reality. And yet, it can turn out that some mathematical theory developed completely within the constraints and rules of mathematics can become applicable to nature. Think of imaginary numbers that are literally 'imaginary' but became a fundamental ingredient in quantum physics and non-Euclidean geometry and higher dimensions are counter-intuitive if we look around us, but it is essential to develop these mathematics before they eventually turn out to be very realistic in describing the relativity of nature in a spacetime universe.

I did a lot of name-dropping in this review, but there are any more artists, scientists, mathematicians, composers, in short many creators in the book. And it is remarkable how with many contemporary names and projects, du Sautoy has been personally involved.

I also recommend to check YouTube where you can find lectures by du Sautoy like the one directly related to this book where he gives a one hour lecture at the Royal Institution (October 2025) where at the end of the lecture the cellist Alia Bekova plays *Nomos Alpha* from Iannis Xenakis, while on the screen du Sautoy simultaneously illustrates how the symmetries of a cube were used in the composition. You can also enjoy him while he's performing with Victoria Gould *I am a strange loop* and you can find the quartet *Four musical theorems and a conjecture*.

Adhemar Bultheel