BELGIAN MATHEMATICAL SOCIETY

Comité National de Mathématique
CNM


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BMS-NCM NEWS: the Newsletter of the
Belgian Mathematical Society and the National Committee for Mathematics

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## BMS-NCM NEWS

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## Letter from the editor



Best wishes to each of you for this New Year.
Welcome
to this January 15, 2007- Issue of our Newsletter!

## Françoise Bastin

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## 1 News from the BMS

Please find enclosed a leaflet for informations (in particular : BMS membership $=$ EUR 20, BMS + EMS membership $=$ EUR 42) and the bank document concerning the renewal of your BMS membership (2007) subscription. Thank you for your attention!

On the label of the envelope (containing this Newsletter), the number $200 x$ indicates your last membership subscription. Thank you for your attention!

## 2 Meetings, Conferences, Lectures

### 2.1 January 2007

A l'occasion du Midterm Review meeting du réseau européen MODNET (model theory and applications) qui se tiendra à Mons le 22 janvier 2007, organisation d'un

## groupe de contact en logique mathématique 21 et 23 janvier 2007 (UMH).

De nombreux chercheurs, postdoctorants et doctorants du réseau MODNET participeront à cette réunion.
Pour toute demande d'information et l'inscription, veuillez consulter en temps utile le site:
http://math.umh.ac.be/logic/seminars.htm ou contacter les organisateurs par email: point@logique.jussieu.fr

### 2.2 April 2007

## 43ste Nederlands Mathematisch Congres

Op donderdag 12 en vrijdag 13 april 2007 wordt in het Gorlaeuscomplex in Leiden onder auspicien van het Koninklijk Wiskundig Genootschap het 43ste Nederlands Mathematisch Congres gehouden, gezamenlijk georganiseerd door de Universiteit Leiden en de Technische Universiteit Delft. Op dit congres zal o.a. de Ostrowskiprijs worden uitgereikt aan Green en Tao (winnaar Fieldsmedaille 2006). Voor het volledige programma en overige informatie zie

For more complete information, see at the address :em www.nmc2007.nl

## Dynamics in Perturbations

on the occasion of the 60th birthday of Freddy Dumortier
Hasselt University
(23-25 april 2007, campus Diepenbeek), and KVAB (26-27 april 2007, Brussels)

Organizers:
Patrick Bonckaert, Magdalena Caubergh, Peter De Maesschalck, André Vanderbauwhede.
Subject:
recent trends in differentiable dynamical systems, in particular geometric, analytic and topological methods in pertubation and bifurcation theory of vector fields. Focus on: singular, Hamiltonian and other perturbations. Essentially the conference will emphasize on results in low dimensions.
Invited speakers include:
Jorge Sotomayor - Universidade de Sao Paulo
Floris Takens - RU Groningen
Carmen Chicone - University of Missouri-Columbia
Hiroshi Kokubu - Kyoto University
Jean-Pierre Francoise - Université P.-M. Curie, Paris VI
Chengzi Li - Beijing University
Christiane Rousseau - Universite de Montreal
Welington de Melo - IMPA, Rio de Janeiro
Henryk Zholadek - University of Warsaw
Douglas Shafer - University of North Carolina at Charlotte
Henk Broer - RU Groningen
Jaume Llibre - Universitat Autònoma de Barcelona
Robert Roussarie - Universite de Bourgogne, Dijon
Carles Simo - Universitat de Barcelona
Yulij Il'yashenko - Cornell University/MCCME Moscow
Homepage:
http://www.uhasselt.be/dysy/dynper/ Email: patrick.bonckaert@uhasselt.be

### 2.3 July 2007

The EMS is a member society of ICIAM (=International Congress on Industrial and Applied Mathematics); please note the

## Congress ICIAM 2007, 16-20 July 2007 in Zurich

See information on the web pages at the address http://www.iciam07.ch/registration
Tuulikki Makelainen, Dept of Mathematics and Statistics POB 68 (Gustaf Hällströmink. 2b), FI-00014 University Helsinki, Finland fax: +358-9-1915 1400
$2.4 \quad 2008$
5ECM, July 14-18, 2008
5th EUROPEAN CONGRESS of MATHEMATICS
Informations can be found at the address http://www.5ecm.nl

## 3 Summary of PhD theses

Last December, UGent,
Nele DE SCHEPPER
(and not Nele DE SCHEEPER) defended her PhD thesis,
Multi-dimensional Continuous Wavelet Transforms and Generalized Fourier Transforms in Clifford Analysis.

Sorry for the mistake.

## Quantifications naturelles projectivement équivariantes

Fabian RADOUX
University of Liège
December, 2006
Promotor: P. Lecomte
For a summary, please see the end of this Newsletter.

## 4 Miscellaneous

### 4.1 From VUB

Prof. Ingrid Daubechies (Princeton) will visit the VUB from March 10 till March 19 , 2007. Contact: pcara@vub.ac.be

## 5 History, maths and art, fiction, jokes, quotations...

The structure of paintings Michael Leyton, Springer Verlag, Wien-New York, 2006 (207 p.), Soft cover, ISBN 3-211-35739-4, 40.00 €.

Michael Leyton is professor at the faculty of psychology and at the DIMACS
 center for Discrete Mathematics and Theoretical Computer Science. His work is as diverse as his affiliation. He publishes in psychology journals, but also in journals about complexity, geometry, computer science, and he is an artist. One may find images of his sculptures, and paintings, on the web and he published musical compositions.

Most of his ideas are the result of an unconventional view of shape and geometry. This idea he has applied or has been applied by others to all kinds of arts, science and technology. His original paper dates back to 1987 [4] in which he explains his Duality-symmetry theorem, and the whole theory is more elaborated in his subsequent book Causality, symmetry, mind [1]. More recently he wrote several books where he is extending and applying his theory to several application fields. In his $A$ generative theory of shape [2], he exposes the "full mathematical foundations" and he applies the theory mainly to CAD/CAM, in Shape as Memory [3] it is applied to architecture and in the present book to the analysis of paintings.

So, what is this geometrical theory that is so universally applicable? This book, as well as several of his other papers or books give an introduction to his views. No mathematics are needed to understand what he means.

First of all he wants to throw away geometry as it existed for the last 3000 years up to and including Einsteins additions. What this traditional geometry is studying are properties that stay invariant under congruence (Klein) or under change of reference frame (Einstein). Leyton's new foundations of geometry are based on a dynamic concept that to each shape one may associate a history. This history can be recovered backward in "time" by removing step by step all asymmetry of the shape which will ultimately result in total symmetry
 represented by a circle (which refers to the egg, origin of everything). In this respect there is some relation to the work of Golubitsky and Stewart [5-6]. Hence it is the asymmetry, not the symmetry that is important because it is the memory that contains the history of the shape, and that is, according to Leyton, exactly what aesthetics are. Thus geometry $=$ memory $=$ aesthetics, and the memory is in the asymmetries. Artworks are maximal memory stores.


Fig. 1: Symmetry-curvature duality theorem: Any section of smooth curve with exactly one curvature extreme has exactly one symmetry axis which terminates at the extreme.

Thus a first step is to define asymmetries. If the curvature is not equal everywhere (like on a circle) then there are extremes in curvature and to each extreme, one may associate a symmetry axis as the collection of points $C$ that are halfway the circular arc between the two tangent points $A$ and $B$ of a circle that touches the curve on either side of the extreme. See figure 1. The process resulting in this asymmetry has evolved along this symmetry axis from a more symmetric situation of the curvature to the present extreme. An ellipse is a deformed circle which is obtained by an internal force pushing it outward along its longest axis or an external force pushing it inward along its short axis.

Thus it is also important to know what in inside (foreground, solid, positive) and what is outside (background, thin, negative). This gives emotional expression to a curve. As one may see on Figure 2, the processes at extrema $M^{+}$and $m^{-}$are penetrating the recipient space, sharpening the extreme, tightening the surrounding space, and facilitate arching. At the other two extrema, the processes are
compressing the recipient space, flattening the extreme, broadening the surrounding space and oppose arching. The $m^{ \pm}$are both inward while the $M^{ \pm}$are outward.


Fig. 2: Four process types: $M^{+}$protrusion, $m^{-}$indentation, $m^{+}$squashing, $M^{-}$resistance

A duality operator switches inside and outside and has the effect that $M^{+} \leftrightarrow m^{-}$ and $m^{+} \leftrightarrow M^{-}$are both dual pairs.

What happens if the dynamics continue at the extrema? For $M^{+}$and $m^{-}$nothing really happens. They continue to be as they are. For $m^{+}$and $M^{-}$things will eventually change: a squashing continues till it indents ( $m^{+} \rightarrow m^{-}$) and a resistance continues till it protrudes $\left(M^{-} \rightarrow M^{+}\right)$.
More interesting are bifurcations. If there is too much resistance against the continuation, bifurcation may be the result. As is shown in Fig. 3 for the bifurcation at $M^{+}$. On the right of Fig. 2, one may see this situation $m^{-} M^{-} m^{-}$ as a bifurcation of what was originally an indentation $\mathrm{m}^{-}$. Similarly, a bifurcation can occur at the other two types of extrema.

The properties seen so far allow Leyton to do a lot of


Fig. 3: Bifurcation at an $M^{+}$: Note that $M^{+} \rightarrow M^{+} m^{+} M^{+}$. analysis on a number of paintings. Curves can be given emotional value and some curves are repeated at different stages of the dynamical process in the same painting which reveals the emotional values and character of the persons represented by the painting.


Fig. 4: Two histories leading to a cusp.

In a last chapter, Layton introduces smoothness-breaking. That is when the smoothness symmetry is broken by a corner or a cusp in the curve. An extreme at a protrusion can evolve into a point "with infinite curvature" which is a corner, and when pushing the process further, it may become a cusp. Another possible history may be an indentation that evolves into such a cusp. See Fig. 4.

Again, these new insights are applied to a full analysis of a number of paintings by Picasso, Raphael, Cézanne, Holbein, Modigliani, Balthus, Ingres, Memling, Gauguin, and an abstract by de Kooning.

The present book is intended to be the first volume of a series because curvature, which is the central theme here, is only one way of memory storage. Subsequent volumes in the series intend to analyse other sources of asymmetry in paintings.

## References

[1] M. Leyton, Symmetry, Causality, Mind, MIT Press, 1992.
[2] M. Leyton, A generative theory of shape, Lecture Notes in Computer Science, Springer, Berlin, 2001.
[3] M. Leyton, Shape as Memory: A Geometric Theory of Architecture. Birkhäuser, Basel, 2006.
[4] M. Leyton, Symmetry-Curvature Duality. Computer vision, graphics, and image processing, 38, 327-341, 1987.
[5] M. Golubitsky, I. Stewart, The symmetry perspective. Progress in Mathematics, vol. 200, Birkhäuser, Basel, 2002.
[6] M. Golubitsky, I. Stewart, Fearful symmetry, is God a geometer?. Penguin, 1993.

## Résumé

On traite dans cet ouvrage de l'existence et de l'unicité de quantifications naturelles projectivement équivariantes au moyen de la théorie des connexions de Cartan.

On démontre qu'une quantification naturelle projectivement équivariante existe pour des opérateurs différentiels agissant entre $\lambda$ et $\mu$-densités si et seulement si la quantification $\operatorname{sl}(m+1, \mathbb{R})$-équivariante correspondante sur $\mathbb{R}^{m}$ existe. Pour cela, on exprime la quantification au moyen d'une formule en termes de la connexion de Cartan normale associée à la structure projective d'une connexion.

On en déduit ensuite une formule explicite pour la quantification naturelle projectivement invariante.

On démontre après la non-unicité d'une telle quantification par le biais de la courbure de la connexion de Cartan normale.

Enfin, on démontre l'existence de quantifications naturelles projectivement équivariantes pour des opérateurs différentiels agissant entre sections d'autres fibrés naturels en transposant la méthode utilisée dans $\mathbb{R}^{m}$ pour analyser l'existence de quantifications $s l(m+1, \mathbb{R})$-équivariantes, méthode liée à l'opérateur de Casimir.

## Summary

One deals in this work with the existence and the uniqueness of natural projectively equivariant quantizations by means of the theory of Cartan connections.

One shows that a natural projectively equivariant quantization exists for differential operators acting between $\lambda$ and $\mu$-densities if and only if the corresponding $\operatorname{sl}(m+1, \mathbb{R})$-equivariant quantization on $\mathbb{R}^{m}$ exists. With this end in view, one writes the quantization by means of a formula in terms of the normal Cartan connection associated to the projective structure of a connection.

One deduces next an explicit formula for the natural projectively equivariant quantization.

One shows the non-uniqueness of such a quantization by means of the curvature of the normal Cartan connection.

Finally, one shows the existence of natural and projectively equivariant quantizations for differential operators acting between sections of other natural fiber bundles transposing the method used in $\mathbb{R}^{m}$ to analyse the existence of $s l(m+1, \mathbb{R})$-equivariant quantizations, this method being linked to the Casimir operator.

