BELGIAN MATHEMATICAL SOCIETY

Comité National de Mathématique CNM

C W N M

NCW Nationaal Comite voor Wiskunde

BMS-NCM NEWS: the Newsletter of the Belgian Mathematical Society and the National Committee for Mathematics

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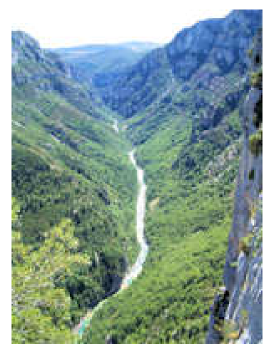

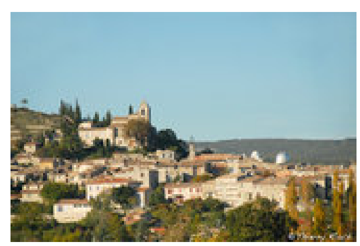

BMS-NCM NEWS

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No 77, March 15, 2010

Letter from the editor

Welcome
to our "March 15, 2010-Newsletter"
Have a nice Spring Time

Françoise

Contents

1 News from the BMS

The next

PhD-day of the BMS

will take place in Brussels (Academy) on

Monday September 13, 2010.

Have a look at the web pages of the BMS (http://bms.ulb.ac.be/cgi/announce.php) or contact one of the member of the organizing committee (Françoise Bastin, F.Bastin@ulg.ac.be; Adhemar Bultheel, adhemar.bultheel@cs.kuleuven.be; Stefaan Caenepeel, scaenepe@vub.ac.be; Philippe Cara, pcara@vub.ac.be; Paul Godin, pgodin@ulb.ac.be; Stefaan Vaes, Stefaan.Vaes@wis.kuleuven.be)

On that day the first *Godeaux Lecture* will be delivered by the *Professor Gilles Godefroy* (Paris 6, Jussieu).

The Godeaux Lecture is organized at least once every two years during a BMS event. These lectures honoring the memory of Lucien Godeaux are organized with the assets of the Belgian Center for Mathematical Studies which were transferred to the BMS after the dissolution of this Center. Lucien Godeaux (1887-1975) was one of the world's most prolific mathematicians (with 644 papers published) and took many initiatives to encourage young mathematicians to communicate their research. He was the founder of the Belgian Center for Mathematical Studies in 1949.

2 Meetings, Conferences, Lectures

2.1 April 2010

Interdisciplinary workshop on
"Sparsity and Modern Mathematical Methods for High Dimensional Data"
April 6-10, 2010, Vrije Universiteit Brussel, Brussels, Belgium

Conference website: http://www.sparsity.be

Topics:
-sparse techniques in inverse problems and compressed sensing (theory, algorithms, applications, ...)
-wavelet-like transforms (shearlets, curvelets, use of non-regular grids, ...)
-statistical multi-resolution modeling and restoration of images (with applications in remote sensing, biomedical imaging, ...)
-analysis of multi-spectral data and the study of art

This workshop will give young scientists in particular the opportunity to present their recent results on new mathematical methods for high-dimensional data and their applications, to a broad audience. In addition, a small number of invited speakers will present their research field in a more general way.

Background:
In recent years exciting new developments in mathematics and computer science have opened up new domains of application for computational mathematics. These developments bring new challenges, for which new approaches and new tools must be developed. These draw not only from traditional linear-algebra-based numerical analysis or approximation theory, but also from information theory, graph theory, the geometry of Banach spaces, probability theory, and more. Often the features, patterns or structures of interest hidden in the data, are typically concentrated on subspaces or manifolds of much smaller dimensions. Even if one has no extra a priori knowledge about which subspace or submanifold might carry the information of interest, the knowledge that it is of much smaller dimension helps in "digging it out". Taking advantage of this underlying sparsity lies at the heart of these new developments. It is also the central tenet of compressed sensing and is presently seeing intense development in inverse problems as well.

The workshop is part of the activities organized on the occasion of the appointment of

Prof. Daubechies
as International Francqui Professor at the VUB (January-June 2010).
http://www.sparsity.be/chair.html

Guest speakers:
Ingrid Daubechies (Princeton University)
Gitta Kutyniok (Universität Osnabrück, Osnabrück)
Javier Portilla (Instituto de Optica, CSIC, Madrid)
David Stork (Ricoh Innovations, Menlo Park)
Pierre Vandergheynst (EPFL, Lausanne)
Dimitri Van De Ville (EPFL, Lausanne)

Organizers:
Ingrid Daubechies (Princeton University), Christine De Mol (ULB, Brussels), Ignace Loris (VUB, Brussels), Benoit Macq (UCL, Louvain) Aleksandra Pizurica (UGent, Gent), Philippe Cara (VUB, Brussels) Ann Dooms (VUB, Brussels), Caroline Verhoeven (VUB, Brussels)

Information:
Conference website and registration: http://www.sparsity.be
Email: info@sparsity.be

Wavelets and Fractals
26-28 April, 2010
Esneux (Liège) , Domaine du Rond-Chêne

This workshop is organized in collaboration with the "Fractal team" of Paris-12 Créteil and with the FNRS contact group "Wavelets and Applications".

Information about this meeting can be found on the web pages at the address

http://www.afaw.ulg.ac.be/scamdec.php

See also the poster at the end of this Newsletter.

Contacts:
Jean-Pierre Antoine, Françoise Bastin (F.Bastin@ulg.ac.be), Christine Demol (demol@ulb.ac.be), Samuel Nicolay (S.Nicolay@ulg.ac.be).

2.2 May 2010

Special lectures by Alain Connes and Pierre Deligne
ULB, May 3rd-4th, 2010

At the occasion of the Doctorate Honoris Causa for Alain Connes and Pierre Deligne at ULB, special lectures for a general audience will take place on
Monday, May 3rd :

14h-15h15	Pierre Deligne (Fields Medal 1978, Wolf Prize 2008)
	La notion d'espace en mathématiques
15h45-17h	Alain Connes (Fields Medal 1982)
	Espace temps, nombres premiers ; deux défis pour la géométrie

Venue : Studio 4 Flagey, Rue du Belvédère 27, 1050 Brussels.
Simultaneous translations in English will be available.

If you wish to attend these talks, you should register on `http://www.nobel175.be`.

Additional lectures for a mathematical audience will be organized on **Tuesday, May 4th** at ULB. More details on these events will be announced on

`http://www.ulb.ac.be/facs/sciences/math`

FNRS group "Functional Analysis"
May 27-28, 2010
Esneux (Liège) , Domaine du Rond-Chêne

Following the tradition, the FNRS group "Functional Analysis" will meet next May (Thursday May 27, Friday May 28). The meeting will take place in the small town of Esneux, in the "Domaine du Rond-Chêne"

The following speakers have already confirmed their participation (alphabetical order):

- R. DEMAZEUX (U. Artois)
- J. JUAN-HUGUET (U. Pol. Valencia)
- M. MAESTRE (U. Valencia)
- J. WENGENROTH (U. Trier)
- P. ZIOLO (A.M. University Poznan)

Contacts: Françoise Bastin (F.Bastin@ulg.ac.be) or Samuel Nicolay (S.Nicolay@ulg.ac.be)

3 PhD theses

Generalized Penrose Transform
Quasi-G-equivariant D-modules and Zuckerman Functor
Rémi Lambert

March 22, 2010 University of Liège

Promoteur: J.-P. Schneiders, ULg

Summary: See the end of the Newsletter

4 Miscellaneous

4.1 Postes de MCF

Appels:

- Un Poste MCF en section 25, profilé "analyse", va être ouvert ce printemps à l'Université de Provence.

  La thématique prioritaire est: Analyse harmonique, Analyse complexe et Analyse fonctionnelle.

  Au-delà de la thématique prioritaire, les dossiers de bon niveau provenant de toutes les branches de l'analyse mathématique seront aussi considérés avec attention.

  Contacts: Alexander Borichev (borichev@cmi.univ-mrs.fr), El Hassan Youssfi (youssfi@cmi.univ-mrs.fr)

- Un Poste MCF en sections 25-26, profilé "analyse", va être ouvert ce printemps à l'Université Paul Sabatier (Toulouse).

  La thématique prioritaire est l'analyse réelle ou harmonique (analyse de Fourier, intégrales singulières, analyse géométrique...), en cohérence avec le recrutement récent de S. Petermichl.

  Le nouveau recruté aura l'occasion d'interagir avec des collègues dans les trois équipes de l'Institut, notamment en analyse complexe, probabilités, géométrie riemannienne, équations aux dérivées partielles.

  Au-delà de la thématique prioritaire, les dossiers de bon niveau provenant de toutes les branches de l'analyse mathématique seront aussi considérés avec attention.

  Contacts: Stefanie Petermichl (stefanie.petermichl@gmail.com), Pascal Thomas (pascal.thomas@math.univ-toulouse.fr)

4.2 From ESF

European research networking programmes EUROCORES

EuroGIGA - Graphs in Geometry and Algorithms

EuroGIGA is the first EUROCORE programme on mathematics, focused specifically on basic but difficult problems summarized under the name "Graphs in Geometry and Algorithms".

Geometric graph and hypergraph structures are at the core of Discrete and Computational Geometry, and simultaneously, they are a crucial tool in applications. Many important and seemingly unrelated problems can be formulated as problems on graphs that include geometric information or implicitly represent it. The studies of geometric graph structures touches upon, and sheds light into various classes of questions, solutions of which are relevant not only in many practically oriented areas of computer science, such as geographic information systems, computer graphics, robotics, and geometric modeling, but also in the applied sciences. Moreover, several problems formulated for geometric graphs belong to the list of prominent open problems in discrete mathematics. Solving such problems is not solely of theoretical interest but will influence future directions of research in the entire area of algorithmic and geometric graphs. EuroGIGA programme will bring together the most advanced techniques from algorithms, combinatorics, algebra, topology, and polyhedral geometry, as well as computer experiments to conquer new frontiers and eventually bring back the fruits of this research to (more applied) algorithmic analysis and to other branches of mathematics which are based on these fundamental questions.

EuroGIGA is supported by 16 funding organisations in Austria, Belgium, Croatia, Czech Republic, Germany, Hungary, Luxembourg, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Switzerland and Turkey. **The Call for Outline Proposals for Collaborative Research Projects within the EuroGIGA has been launched with closure on 29 March 2010, while the expected deadline for full proposals is 22 June 2010.** The expected kick-off of the programme is spring 2011.

Collaborative Research Projects are invited to apply within the following research topics:

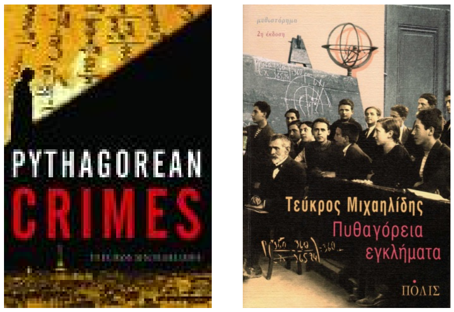
- Drawing graphs;
- Geometric representations of graphs;
- Skeletal structures and spatial decompositions;
- Combinatorics of discrete point sets;
- Graphs of polytopal structures.

See the web pages at the address

http://www.esf.org/activities/eurocores/programmes-in-preparation/eurogiga.html

5 History, maths and art, fiction, jokes, quotations...

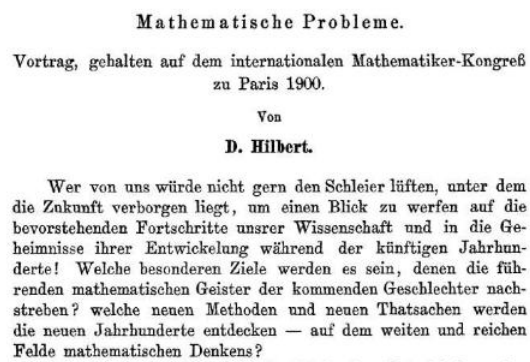
Pythagorean crimes. Tefcros Michaelides, Parmenides, 2008, 272 p., ISBN: 978-1-930972-27-8.



Cover of English and Greek version

This is the English translation of the original novel that was published in Greek in 2006. The author, a mathematics professor at Athens College in Greece, is probably best known in his own country, for having translated several novels of the "popular science / mathematical fiction"-type into Greek (*A Short History of Nearly Everything* (B. Bryson), *The Parrots Theorem* (D. Guedj), *Timescape* (G. Benford), *Mobius Dick* (A. Crumey), and several others). He also published several texts on the influence of mathematics on culture, literature, and society from ancient times to science fiction and from China to the USA. This English translation brought him an international breakthrough.

From the title one might expect a crime thriller, and indeed, the opening chapter starts by introducing the narrator of the story, Michael Igerinos, who is called by the police to come to the home of his best friend, Stefanos Kandartsis, who is found poisoned in his bed. But then, Michael begins an account of his relation with Stefanos, starting with their first encounter at the international congress of mathematics in August 1900 in Paris. Both of these main characters are Greek, and both are mathematicians. Michael is from a rich aristocrat family and studied in Göttingen (in those days the home of Klein, Hilbert, Carathéodory and others) while Stefanos has a much poorer background, and he studied in Paris. It was at that conference that Hilbert formulated his 23 open problems, the mathematical challenges for the 20th century. You may find them on wikipedia, or in much more detail, together with the quest for their solution in the book by Ben H. Yandell: *The honor's class: Hilbert problems and their solvers* (A K Peters, 2003). The second problem about the consistency of an axiomatic system (that got an answer in Gödel's incompleteness theorem, published in 1931) plays a central role (the time-span of the novel is 1900-1931).


Mathematische Probleme.

Vortrag, gehalten auf dem internationalen Mathematiker-Kongreß zu Paris 1900.

Von

D. Hilbert.

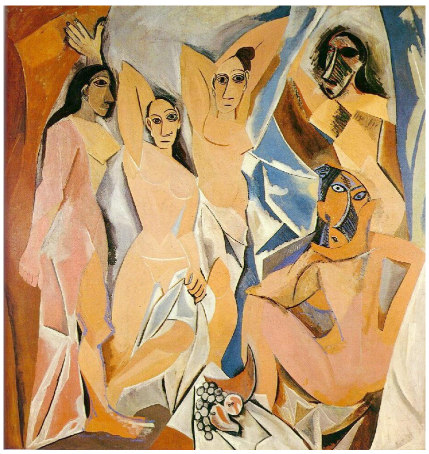
Wer von uns würde nicht gern den Schleier lüften, unter dem die Zukunft verborgen liegt, um einen Blick zu werfen auf die bevorstehenden Fortschritte unsrer Wissenschaft und in die Geheimnisse ihrer Entwickelung während der künftigen Jahrhunderte! Welche besonderen Ziele werden es sein, denen die führenden mathematischen Geister der kommenden Geschlechter nachstreben? welche neuen Methoden und neuen Thatsachen werden die neuen Jahrhunderte entdecken — auf dem weiten und reichen Felde mathematischen Denkens?


Henri de Toulouse-Lautrec

In the early 20th century, Paris was a swinging place-to-be and a center of the Western world. There was the *Exposition Universelle* in 1900, and it attracted many artists, among which the young Pablo Picasso and his *bande* of close friends such as Guillaume Apollinaire, Max Jacob, André Salmon, and also Maurice Princet, who has been called the "mathematician of cubism". Two year earlier, Emile Zola published his *J'accuse* about the Dreyfus case, Sarah Bernhardt performs on stage, and there is of course the infamous night-life that gave Paris its reputation. Henri de Toulouse-Lautrec was hanging out in the mondain *Moulin Rouge* with Jane Avril dancing the *can-can*, Lautrec's posters are all over Paris, and Aristide Bruant's cabaret *Le Mirliton* is living its high days, etc.

So, the novel drifts off for about three quarters of the book into a sketch of society, with a lot of mathematical gossip, the *petites histoires* of mathematics, but also, as the two protagonists discuss mathematics among themselves or explain it to others, the reader is instructed as well.

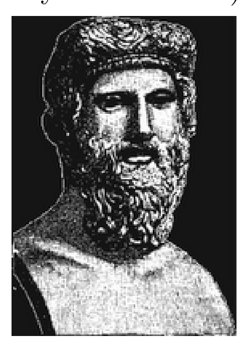
However, the amount of facts, events, theories, and people that are dumped in these 180 pages is somewhat of an overdose. The author knows very well how to tell a story, and it is very well written indeed, but there is just too much of it. For a mathematician, who is more or less familiar with the basics, this may be an interesting read, since the mathematicians are presented as very human, of flesh and blood, with their idiosyncrasies, their physical appearance, their social behaviour, their rivalry and their sharp minds. However, I doubt that, without further reading or help, a non-mathematician can catch all what has been intended. There is a glossary at the end of the book (especially compiled for the English edition) of 40 pages with some 220 entries giving short descriptions of people and facts.



Les Mademoiselles d'Avignon

There is not only the mathematics though, also Pablo Picasso and his gang, who gather in Montmartre places like the *Zut*, and the *Le Lapin Agile* play an important role. With this group they discuss the tiling of the plane and non-euclidean geometry for example. This is related to the representation of form, much to Picasso's interest since he is struggling with his famous painting *Les Mademoiselles d'Avignon*, which was only finished in 1907, but Michael and Stefanos can have a peek at the canvas still hidden against the wall. At the same time, the relations between the members of the group is carefully sketched.

But then the story takes a turn. Michael has to go back to Greece to take care of family affairs after his father died. He marries (more a business agreement with a bilateral open relation), but it gives the author the opportunity to send them on a honey-moon trip to Paris in 1905 where Michael can meet his old friends again. When his wife leaves him to start a relation with Stefanos, Michael buys a prostitute off from a human trafficker, but also she leaves him for Stefanos. Then there is a part on modern Greek history, when both friends have to fight while Greece engages in the first World War: Stefanos in the front line, Michael in the Headquarters. This part seems to be of little interest to the whole story (except perhaps when you are Greek).



Hippasus of Metapontum

And then there is the crime story. Gradually Michael gets more and more discredited for the death of Stefanos and is finally sentenced for murder. However, thanks to an expensive lawyer, he is declared not guilty in appeal and released from prison. But then, there is a final twist to the story...

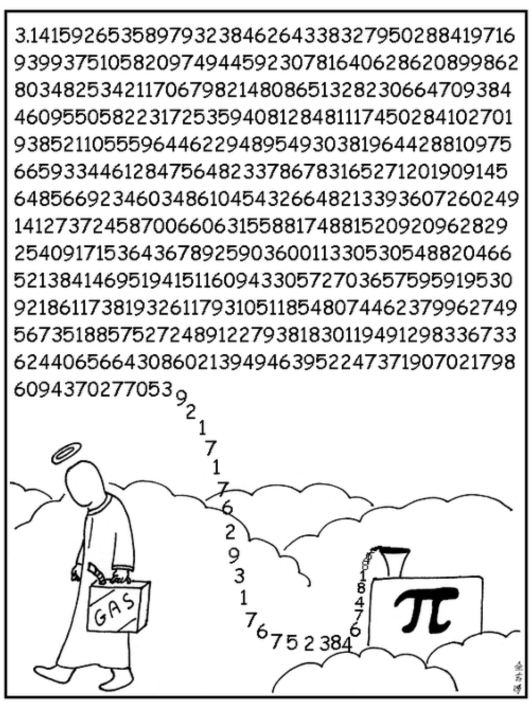
So, then what has Pythagoras got to do with all this? It is known that the Pythagoreans was more like a sect with strict rules of secrecy and that one of their beliefs was that everything was numbers (i.e., integers). So the problem of squaring the circle was a tedious point to discuss, the diagonal of a square with side 1, being $\sqrt{2}$. In three short interludes, the author briefly tells the story of Hippasus of Metapontum (ca. 500 BC), a disciple of Pythagoras, who had allegedly discovered that $\sqrt{2}$ is irrational, which was considered religious heresy, and the legend says that he was murdered so that he could not disclose his findings because that would mean the end of the whole Pythagorean system, a senseless thing to do since history takes it course anyway. So there is some relation between this novel and *Pythagoras' revenge* by A. Sangalli that was reviewed in the January 2010 issue of this Newsletter (no. 76).

Adhemar Bultheel

new pi-trivia

Did you know ...

- ... that today is π-day? Why? Because in America they write 3/14 for the date of today March 14, and 3.14 is an approximation to the number π.
- ... that recently, on December 31st 2009, a new world record in the computation of digits of π was set by Fabrice Bellard?
  A total of 2 699 999 990 000 decimal digits were computed (on a single desktop computer). The calculation was done using the Chudnovsky series:
  $$\frac{1}{\pi} = 12\sum_{n=0}^{\infty}(-1)^n\frac{(6n)!(A+Bn)}{(n!)^3(3n)!C^{3n+3/2}}$$
  with $A = 13591409, B = 545140134, C = 640320$.
  This series gives 14 decimal digits accurately per term. Computation of the binary digits took 103 days.



- ... that the last two of the sixteen digits of π that Isaac Newton computed in 1665-1666 were wrong? Newton used the following integral
  $$\int_0^{1/4}\sqrt{x-x^2}\,\mathrm{d}x = \frac{\pi}{24} - \frac{\sqrt{3}}{32}$$
  He calculated an approximation to the integral using his binomial series.
  Newton noted: "I am ashamed to tell you to how many figures I carried these computations, having no other business at the time".
- ... that the Fibonacci numbers can be used to calculate approximations to π? If we denote the terms of the Fibonacci-sequence by F_n : $F_0 = 1, F_1 = 1, F_2 = 2, F_3 = 3$ and $F_{n+2} = F_{n+1} + F_n$, then is is possible to prove that
  $$\sum_{n=0}^{\infty}\arctan\frac{1}{F_{2n}} = \frac{\pi}{2}$$
  This is a consequence of the equality:
  $$\sum_{i=0}^{n}\arctan\frac{1}{F_{2i}} = \arctan F_{2n+1}$$
- ... that in Ramanujan's second notebook the following beautiful formula can be found?
  $$\cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{1+\cfrac{5}{\cdots}}}}}} +$$
  $$1 + \frac{1}{1\cdot 3} + \frac{1}{1\cdot 3\cdot 5} + \frac{1}{1\cdot 3\cdot 5\cdot 7} + \ldots$$
  $$= \sqrt{\frac{\pi \mathrm{e}}{2}}$$
- ... that Andriy Tychonovych Slyusarchuk, a Ukrainian neurosurgeon, medical doctor and professor, claimed in June 2009 to have set a new record by memorizing the first 30 million places of π, which were printed in 20 volumes of text? Although he did not recite all 30 million digits, he was able to recite sequences that were randomly selected by a jury from within the first 30 million places of π.

- ... that in a scene of the very popular Disney-movie High School Musical a series involving π due to Ramanujan can be seen on the blackboard? One of the students asks the teacher: "Shouldn't the second equation read sixteen over pi?". The teacher then uses her calculator to verify the sum, and corrects her mistake:

- ... that there is a special name for the study of the creation and use of mnemonic techniques to remember a span of digits of π? We call it piphilology. (Note the combination with the golden ratio ϕ ;-) In piphilology *piems* are very important. Piem is a contraction of pi and poem. Piems are poems that represent π in a way such that the length of each word (in letters) represents a digit. Here is a nice one:

  How I wish I could enumerate
  pi easily, since all these bullshit
  mnemonics prevent recalling any
  of pi's sequence more simply.

- ... that there are many pandigital approximations to π? Pandigital means that in this approximation all digits from 1 to 9 appear exactly once. Here is an example. The number

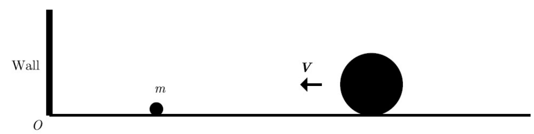
$$3 + \frac{1 - (9 - 8^{-5})^{-6}}{7 + 2^{-4}}$$

  gives an approximation for π that is correct to 9 decimal places. (Note that there is a much better one for e:

$$\left(1 + 9^{-4^{7\cdot 6}}\right)^{3^{2^{85}}}$$

  is correct to 18457734525360901453873570 decimal digits.)

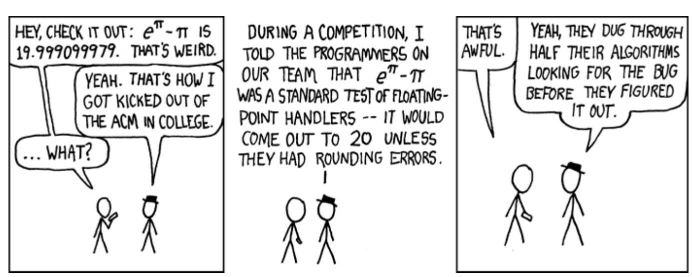
- ... that in the 21st episode of the Simpsons' fourth season ("Marge in Chains") Kwik-E-Mart operator Apu Nahasapeemapetilon testifies in court that he is able to recite 40 000 decimal digits of π? He correctly notes that the 40 000th digit is the number one. Apparently the episode's writers prepared for this scene by asking the National Aeronautics and Space Administration for the number at the 40 000th decimal place of π, and NASA sent them back a printout of the first 40 000 digits.
- ... that Buffon's Needle experiment isn't the only weird way to estimate the value of π? Counting collisions in a simple dynamical system with two balls is another one. But this method is an entirely deterministic one and it can be used to estimate π to any accuracy. This is a picture of the situation:



  The wall is assumed to be absolutely elastic. Push the big ball towards the small ball very fast. If the mass of the big ball is 100^N times the mass of the small ball, then the number of collisions will be a number with $N+1$ digits. The first N digits of this number will be equal to the first N digits of π (starting with 3).
- ... that the following short C program calculates π to 15 000 digits?

```
a[52514],b,c=52514,d,e,f=1e4,g,h;
main(){for(;b=c-=14;h=printf("%04d",
e+d/f))for(e=d%=f;g=-b*2;d/=g)
d=d*b+f*(h?a[b]:f/5),a[b]=d%-g;}
```

- ... that $e^{\pi\sqrt{163}}$ and $e^\pi - \pi$ are well-known almost integers? (http://www.xkcd.com)



(Paul Levrie)

WAVELETS & FRACTALS

April 26-28, 2010
Domaine du Rond-Chêne, Esneux
Liège Belgium

Main Speakers

- **A. Arneodo**
- **I. Daubechies**
- **M. Dekking**
- **J. Schmeling**

Organizers

J.-P. Antoine
J.-M. Aubry
F. Bastin
C. De Mol
S. Jaffard
S. Nicolay
S. Seuret

Joint meeting with the SCAM Paris XII and the FNRS contact group "Wavelets and applications"

Information and registration
http://www.afaw.ulg.ac.be/scamdec.php

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SCAM

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Twistor theory and Penrose Transform

The twistor theory began in the late 1960's with the publication of some R. PENROSE's papers [6, 7, 8] in which he introduced the notion of twistors and twistor geometry. The fundamental idea given by R. PENROSE in his seminal paper entitled "*twistor algebra*" [6] is to replace the usual background spacetime used for the decription of many physical phenomena, by a new background space called "*twistor space*". The physical objects (spacetime, particles, fields, etc.) and the equations describing the phenomena are reinterpreted in this new twistor space, with the intent to get new insight into them. By analogy, R. PENROSE's idea is similar to introducing the momentum space and using the Fourier transform to translate back and forth from spacetime to momentum space. The name "*twistor theory*" is derived from the Robinson congruence which is the natural realization of a twistor (see [6]).

There are several aspects to the twistor program initiated by R. PENROSE. First, it successfully gave a new insight into many nonlinear classical field equations and even a new point of view for the classical linear field theories. New families of solutions to the nonlinear equations have been found and sometimes with the discovery of complete classification of the solutions. This approach was of interest to theoretical physicists to study solutions of equations in quantum field theory, general relativity theory and in some more recent attempts to quantize gravity. The mathematics of the twistor theory has the specific quality to use classical mathematical tools as well as others more modern. For example, this theory is based on some works from the 19^{th} century by H.G. GRASSMANN, J. PLÜCKER and F. KLEIN in projective and algebraic geometry and its formalism uses sheaf theory, sheaf cohomology invented by J. LERAY in 1949 and some works of J.-P. SERRE and H. CARTAN whose ideas considerably helped to reinterpret classical phenomena in terms of the new language.

Using these twistor spaces, R. PENROSE [6] showed in 1967 that the solutions of some important physical conformally invariant differential equations on $\mathbb{C}^4$ (massless fields in particular) could be expressed as contour integrals of holomorphic functions in twistor space. He also noticed that the functions appearing in these expressions were not unique, without however precisely determining the nature of the liberties they enjoyed. In 1981, M. EASTWOOD, R. PENROSE and R. O. WELLS solved this problem in [3] identifying this liberty as being the one of a representative of a Čech cohomology class. The resulting isomorphism between sheaf cohomology groups of the complex projective 3-space and solutions of conformal invariant equations over the complexified and compactified Minkowski spacetime has become known as the Penrose Transform.

Since the works of M. EASTWOOD, R. PENROSE and R. O. WELLS, the original Penrose transform has been generalized in several directions and studied in different contexts. Independently of R. Penrose, W. Schmidt studied in 1967 the representations of Dolbeault cohomology groups using methods similar to the original Penrose transform (see [9, 10]). A generalization has been detailed by R. J. Baston and M. G. Eastwood in [2] where the conformal group $\mathbb{C}^4$ is replaced by an arbitrary semisimple complex Lie group G. The generalized Penrose transform obtained in this way maps Dolbeault cohomology groups of homogeneous vector bundles on a complex homogeneous manifold Z to solutions of invariant holomorphic differential equations on a complex submanifold of Z. Using the classical Lie group decomposition in terms of simple roots of the associated Lie algebra, Bott-Borel-Weil theorem and the Bernstein-Gelfand-Gelfand resolution, the authors describe an algorithmic way to compute explicitely this generalized Penrose transform.

Since solutions of differential equations on manifolds and cohomology classes are part of the bases of this generalized Penrose transform, the theory of sheaf cohomology and $\mathcal{D}$-modules was perfectly suited to it for its modern study. In 1996, A. D'Agnolo and P. Schapira used this theory to generalize and study the R. Penrose's correspondence in [1]. More recently, C. Marastoni and T. Tanisaki in [5] studied the generalized Penrose transform between generalized flag manifolds over a complex algebraic group G, using M. Kashiwara's correspondence (see [4]) between quasi-G-equivariant $\mathcal{D}_{G/H}$- modules and some kind of representation spaces which are ($\mathfrak{g}$, H)-modules (loosely, they are complex vector spaces endowed together with an action of the Lie algebra $\mathfrak{g}$ associated to G and a action of H which are compatible in some way) when H is a closed algebraic subgroup of G. The subject of this thesis is inspired by this correspondence.

Outline

The question studied in this PhD. thesis was introduced to us by J.-P. Schneiders. When H is a closed algebraic subgroup of a given complex algebraic group G, M. Kashiwara has established in [4] a correspondence of categories between (derived) category of quasi-G-equivariant $\mathcal{D}_{G/H}$-modules and (derived) category of ($\mathfrak{g}$, H)-modules. Since the R. Penrose's integral transform was described and generalized on the quasi-equivariant $\mathcal{D}$-modules side by C. Marastoni and T. Tanisaki in [5], an interesting problem was to determine the corresponding algebraic transform in terms of functors defined on some representation spaces via M. Kashiwara's equivalence and to explain a method to compute it.

To achieve this contribution, we proceed in three steps. Since the generalized Penrose transform for $\mathcal{D}$-modules is the composition of an inverse image functor and a direct image functor on the $\mathcal{D}$-module side, we first describe the algebraic analogs of these functors. More explicitely, if $H \subset K$

are two closed algebraic subgroups of a given complex algebraic group G, we show that the $\mathcal{D}$-module inverse image functor associated to the projection $g : G/H \to G/K$ is equivalent to the forgetful functor from the category of $(\mathfrak{g}, K)$-modules to the category of $(\mathfrak{g}, H)$-modules. We also show that the derived direct image functor $\mathbf{D}g_*$ on the $\mathcal{D}$-module side corresponds (up to a shift) to the algebraic Zuckerman functor $\mathrm{R}\Gamma_H^K$ which maps $(\mathfrak{g}, H)$-modules to $(\mathfrak{g}, K)$-modules (more precisely, objects of the derived categories).

The next step is to obtain an analog of the Bott-Borel-Weil theorem for computing the Zuckerman's functor image of "basic objects" of the category of $(\mathfrak{g}, H)$-modules. More precisely, we compute explicitely the image by the derived Zuckerman functor $\mathrm{R}\Gamma_H^K$ of generalized Verma modules $M_H(\lambda)$ associated to weights λ which are integral for $\mathfrak{g}$ and dominant for the Lie subalgebra of $\mathfrak{g}$ corresponding to H. Those generalized Verma modules are the objects which correspond via M. Kashiwara's correspondence to the $\mathcal{D}$-modules which generate the Grothendieck group of the category of quasi-G-equivariant $\mathcal{D}_{G/H}$-modules of finite length.

Finally, we describe a method to analyse the image of such generalized Verma modules $M_H(\lambda)$ by the algebraic transform which corresponds to the so-called generalized Penrose transform. This analysis is inspired by the Bott-Borel-Weil theorem and Bernstein-Gelfand-Gelfand resolution used by R. J. Baston and M. G. Eastwood in [2].

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