The universality problem in mathematics

Mirna Džamonja

(Paris)

We discuss the following problem: let K be a class of objects equipped with a reflexive transitive relation $\leq$. For example, K could be the class of all graphs of a given infinite size k, with the relation L $\leq$ L' being that there exists an order-preserving embedding from L to L'. We discuss methods that have been developed for recognising situations when K has or does not have a dominant element, meaning an element X^* such that for all $X \in K$ we have $X \leq X^*$. Such an element is called a universal element of K. For example, when K is the class of all countable graphs, there is a universal element in K, namely the random graph. Already in this example and at the next infinite cardinal, the situation is much more complex and the answer is independent of the axioms of set theory.