PhD Day 2024 of the Belgian Mathematical Society

University of Antwerp, Belgium

May 24, 2024

Schedule of the talks

	G.010	**G.004**	**G.006**
13:45	Lathouwers	Gómez Cobos	Simoens
	De Weerdt	Shaimerdenov	Robinson
	Loué	Carletti	Baum
15:15	Matthys	Belrhazi	Bosco
	Donvil	Pilatte	Bizjak
	Chopra	Winter	

Titles and Abstracts

- **Maarten Lathouwers** (KULeuven), *Twisted conjugacy growth in virtually abelian groups.*

  For an endomorphism f of a group G define the twisted conjugacy relation on G by x y if and only if there is a z such that $x = zyf(z)^{-1}$. The equivalence classes are called the f-twisted conjugacy classes of G. Any finitely generating set S of a group G induces a norm on the group elements. We use $B_S(r)$ to denote all elements of norm less than r. The twisted conjugacy growth is the number of twisted conjugacy classes that intersect these sets $B_S(r)$.

  In this talk, I introduce the above concepts with Z^2 as example in mind. As a final result, I discuss the twisted conjugacy growth in virtually abelian groups using some visualisations of the problem in Z^2.

- **Lore De Weerdt** (KULeuven), *Nielsen numbers of n-valued maps on infra-nilmanifolds.*

  We prove an averaging formula for the Nielsen number of an n-valued map on an infra-nilmanifold, by establishing a link with the coincidence numbers of single-valued maps on a covering nilmanifold.

- **Alex Loué** (UCLouvain), *Triangle presentations.*

  Interesting families of groups acting on 2-dimensional triangle complexes can be constructed using the very simple combinatorial datum of a "triangle presentation". After spending some time on motivation and context for searching such data, I will explain how I was able to construct a new family of exotic lattices in A2 buildings and another similar family of groups by describing highly symmetric triangle presentations.

- **David Santiago Gómez Cobos** (UGent), *L^p-bounds in Safarov pseudo-differential calculus on manifolds with bounded geometry.*

  Pseudo-differential operators on smooth manifolds are defined classically by means of local coordinates using the well-known theory for R^n. Since we would like to have invariance under change of coordinates this will naturally imply some restrictions on the classes that we can construct, particularly when we are faced with the restriction $\rho > 1/2$ on the classes $S^m_{\rho,\delta}$. Safarov introduced a pseudo-differential calculus on smooth manifolds using a linear connection ∇ in order to obtain a global symbol. Using this construction and assuming that the connection is symmetric he was able to weaken the aforementioned restriction to $\rho > 1/3$. We will introduce the Safarov pseudo-differential calculus and we will present a result on L^p - L^p estimates for this calculus.

- **Yerkin Shaimerdenov** (UGent), *Critical Sobolev type (Improved Hardy) inequalities, identities, and their applications.*

  Let us recall the cylindrical extended weighted Hardy type inequality [2]: Let $\alpha \in \mathbb{R}$, $x = (x', x'') \in \mathbb{R}^N \times \mathbb{R}^{n-N}, 2 \leq N \leq n$ and $1 < p < N$. Then we have

  $$\left\| \frac{f}{|x'|^{\frac{\alpha}{p}}} \right\|_{L^p(\mathbb{R}^n)} \leq \frac{p}{N-\alpha} \left\| \frac{x' \cdot \nabla_N f}{|x'|^{\frac{\alpha}{p}}} \right\|_{L^p(\mathbb{R}^n)}, \tag{1}$$

  where $|x'|$ is the Euclidean norm on $\mathbb{R}^N$ and ∇_N is the standard gradient on R^N. The constant $\frac{p}{N-\alpha}$ is sharp for all $f \in C_0^\infty(\mathbb{R}^n \backslash \{x' = 0\})$. When $\alpha = 0$ inequality (1) implies the cylindrical extension of the Sobolev type inequality in [3] when $\alpha = p$ and the Cauchy-Schwarz inequality is used on the right-hand side it implies the Cylindrical Hardy inequality in [1].

  This talk demonstrates the critical case $\alpha = N$ of the inequality (1) with a sharp constant. Moreover, we show identities and higher-order versions and applications. Interestingly, higher-order versions generate Stirling numbers of the second kind.

  This talk is based on joint research with my supervisors Michael Ruzhansky (Ghent University) and Nurgissa Yessirkegenov (SDU University and Institute of Mathematics and Mathematical Modeling, Kazakhstan).

  [1] Badiale M., Tarantello G. A Sobolev–Hardy inequality with applications to a nonlinear elliptic equation arising in astrophysics, *Arch. Ration. Mech. Anal.*, **163** (2002), 259–293.

  [2] Kalaman M. Functional inequalities on Lie groups and applications., *Master's thesis, SDU University,* (2023).

  [3] Ozawa T., Sasaki H. Inequalities associated with dilations, *Commun. Contemp. Math.*, **11(2)** (2009), 265—277.

- **Lorenzi Carletti** (ULB), *The optimal constant for critical Sobolev embeddings on manifolds.*

  Let (M, g) be a compact Riemannian manifold of dimension $n \geq 3$, and $k < n/2$ an integer. In the Euclidean setting, there is an optimal constant $K_0(n,k)$ such that $\left(\int_{\mathbb{R}^n} |u|^{2^\sharp}\right)^{2/2^\sharp} \leq K_0^2 \int_{\mathbb{R}^n} |\nabla^k u|^2$ for all $u \in C_c^\infty(\mathbb{R}^n)$, where $2^\sharp = \frac{2n}{n-2k}$. We show that this constant remains optimal for the critical Sobolev embedding $H^k(M) \subset L^{2^\sharp}(M)$: There exists B_0 such that $\|u\|^2_{L^{2^\sharp}(M)} \leq K_0^2 \int_M |\nabla^k u|^2 + B_0 \|u\|^2_{H^{k-1}(M)}$ for all $u \in H^k(M)$. In this talk, I will present the main ideas of the proof. As it turns out, this leads to studying the pointwise blow-up of a sequence of positive functions $(u_\alpha)_{\alpha\geq 1}$ satisfying $(\Delta_g+\alpha)^k u_\alpha = u_\alpha^{2^\sharp-1}$ in M, for all α.

- **Robin Simoens** (UGent),*Coding theory: into the quantum world.*

  Classical error-correcting codes were introduced in 1950 by Richard Hamming and are now well-developed. They are built upon the idea of adding redundant information to protect against errors. In 1995, Peter Shor showed that this idea can be carried into the quantum world, giving rise to quantum error-correcting codes, which play a central role in the realisation of quantum computers. In this talk, I will give an introduction to both classical and quantum codes, pointing out their differences and similarities.

- **Eileen Robinson** (ULB), *Path eccentricity in graphs.*

  The central path problem is a variation of the single facility location problem. The aim is to find, in a given connected graph G, a path P minimizing its eccentricity, which is the maximal distance from P to any vertex of the graph G. The path eccentricity of G is the minimal eccentricity achievable over all paths in G.

  Gómez and Gutiérrez asked if there is a relation between the consecutive ones property and the path eccentricity of a graph. In this talk we characterize graphs having variations of the consecutive ones property and show that they have path eccentricity at most 2.

- **Carole Baum** (ULiège), *Robustness under missing data: a comparison with special attention to inference.*

  Missing value imputation is a highly studied topic. A plethora of techniques have been proposed over the years to find suitable values to replace missing data. Nowadays, imputation techniques are widely in use, but a large-scale comparison of these methods - and especially in terms of their robustness against outliers - seems to be missing. During a first attempt to fill this gap, we evaluate a large selection of imputation techniques, involving classic and robust procedures, by means of a simulation study with continuous data and different configurations of missing data and outliers. To evaluate the imputation capability and robustness of the imputation techniques we computed the error between the original and the imputed values. However, often, the main concern is on the analysis that is performed after imputation. Therefore, in the second phase of our research, we evaluated the inferences and predictions made by different robust regression methods combined with an imputation technique in a simulation study. Both rowwise and cellwise outliers were generated, so we considered in the evaluation rowwise robust regression techniques as well as cellwise robust regression techniques. To evaluate the combined regression and imputation strategies in terms of inference capability, we measured the bias and variance of the estimated regression coefficients.

- **Joren Matthys** (KULeuven), *Residual Finiteness Growth.*

  Residual Finiteness Growth attempts to measure the extent to which a group is residually finite. A group is residually finite if for all non-trivial elements we can find a homomorphism to a finite group such that the image of that element is non-trivial. The Residual Finiteness Growth then estimates the size of the smallest possible finite group achieving this in terms of the word norm of the chosen element.

  Since its birth in 2010, mathematicians have been able to make estimates for multiple classes, however, these estimates have often been inaccurate. Accurate results have been mostly missing, untill our progress in 2023, when we were able to establish exact results for all virtually abelian groups.

We give an overview of the current status.

- **Milan Donvil** (KULeuven), *W*-superrigidity for cocycle twisted group von Neumann algebras.*

  By representing a countable group G on the Hilbert space $\ell^2(G)$ by left translation operators, one generates the group von Neumann algebra $L(G)$, which can also be viewed as the completion of the complex group ring $C[G]$ in a weak topology. Typically, in the transition from G to $L(G)$ most information about G is lost. However, for specific classes of groups, $L(G)$ completely remembers the group G in the following sense: if $L(G)$ is isomorphic to $L(H)$ for another group H, then G is isomorphic to H. In this case, we say that G is a W*-superrigid group. In a recent joint work with Stefaan Vaes, we generalize W*-superrigidity for groups in two directions. Firstly, we provide a class of groups G for which W*-superrigidity holds in the presence of a twist by a 2-cocycle: if H is any countable group and ω and η are any 2-cocycles on G and H respectively, we have that the twisted group von Neumann algebras $L_\omega(G)$ and $L_\eta(H)$ are isomorphic if and only if the pairs (G, ω) and (H, η) are isomorphic. Secondly, for this same class of groups, the superrigidity also holds up to virtual isomorphism.

- **Alakh Dhruv Chopra** (UGent), *Well-quasi-orders of finite trees and their strength.*

  The talk would be an introduction to the theory of well-quasi-orders and an associated ordinal invariant called its maximal order type, focusing on both classical orders (for example: Higman's ordering, Kruskal's tree order), and original work on finite trees under homomorphic embeddings and their connection to transfinite sequences.

- **Malika Belrhazi** (UAntwerpen), *Geodesic extension of mechanical systems with nonholonomic constraints.*

  For a Lagrangian system with nonholonomic constraints (L, D), the equations of motion follow from the so-called Lagrange-d'Alembert principle. It is well-known that this is not a purely variational principle (for the Lagrangian functional associated to L), since it only makes use of variations that are required to follow the constraints. The resulting equations, the so-called Lagrange-d'Alembert equations, are therefore not simply the Euler-Lagrange equations of L. In this talk we will construct extensions of these equations of motion to sets of second-order ordinary differential equations. This is what we will call a SODE extension.

  In the case of a purely kinetic Lagrangian, we will derive the conditions under which the solutions of the Lagrange-d'Alembert equations are geodesics of a Riemannian metric, while preserving the constrained Lagrangian. We call this a geodesic extension. We interpret the algebraic and PDE conditions of this problem as infinitesimal versions of the relation between the nonholonomic exponential map and the Riemannian metric.

  We discuss the special case of a Chaplygin system with symmetries and we end the talk with a worked-out example.

- **Cédric Pilatte** (University of Oxford), *Recent progress on factoring with quantum computers.*

In 1994, Shor introduced his famous quantum algorithm to factor integers and compute discrete logarithms in polynomial time. In 2023, Regev proposed an improved, multi-dimensional variant of Shor's algorithm, that is more efficient in that it requires much fewer quantum gates. However, this new algorithm relies on a number theory conjecture about the relations between small primes in the multiplicative group modulo N. We prove a version of this conjecture using tools from analytic number theory, thus obtaining an unconditional proof of correctness of these multi-dimensional quantum algorithms.

- **Leon Winter** (UCLouvain), *Traces and extensions of gauge covariant Sobolev spaces.*

  Gauge covariant Sobolev spaces are spaces of sections of a vector bundle for which a weak covariant derivative satisfies a condition of integrability. They are the natural framework for various scalar fields in Yang–Mills equations.

  As for vector-valued Sobolev functions, the sections contained in gauge covariant Sobolev spaces are defined up the a set of measure zero. In particular, it is not clear how to translate a (strong) boundary value problem, whose solution is a smooth function, into a weak boundary value problem, whose solution is an element of a Sobolev-type space.

  For gauge covariant Sobolev sections, one can show that it is possible to give a coherent meaning the the boundary value, called the trace of the function, which is itself an element of a fractional gauge covariant Sobolev space. Conversly, given an element of a fractional gauge covariant Sobolev space, one can show that there exists a section in the (not-fractional) gauge covariant Sobolev space whose trace is the previously given element. This gives a complete characterisation of the trace space for gauge covariant Sobolev spaces.

  In degenerate cases, this result naturally reduces to the trace theorem due to Gagliardo which deals with traces for vector-valued Sobolev functions.

- **Giovanni Bosco** (UMons), *Local geometric Galois representations.*

  Given an abelian variety A/Qp one can construct a p-adic representation of the absolute Galois group of Qp, namely its p-adic Tate module. We are interested in the inverse problem in the case of potential good reduction. Such representations are known to be potentially crystalline and are studied through the use of p-adic Hodge Theory. I will give a first classification in the case of elliptic curves over Q3.

- **Luka Bizjak** (ULB), *Weight structures on the category of K-motives.*

  We will present the notion of the weight structure on the category of K-motives and explain how we can use weight structures to get a more "concrete" description of K-motives. In particular we explain the notion of the weight complex functor, which enables us to study the category of K-motives as a category of bounded complexes over an additive category. We will start with a brief introduction to the notion of a motive, then we will move toward weight structures and explain how one can use them in the realm of K-motives.